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Model World: The Great Debate—MAUT Versus AHP

Saul I. Gass
Robert H. Smith School of Business, University of Maryland, College Park, Maryland 20742,
sgass@rhsmith.umd.edu

The analytic hierarchy process has gained wide acceptance among academics and practitioners.

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"The theory of decision analysis (DA) is designed to help the individual make a choice among a set of *prespecified* alternatives" (Keeney and Raiffa 1976, vii). Simply put, the basic DA problem is, How should a decision maker select from a given set of competing alternatives that are evaluated against conflicting objectives? Because operations research (OR) is the science of decision making, OR researchers and practitioners have developed a wide range of methods for resolving the basic DA problem and its variations. They base some of these methods on von Neumann and Morgenstern's (1947) expected utility theory (EUT) and Savage's (1954) subjective expected utility theory (SEUT), with extensions to multiattribute utility theory (MAUT) and multiattribute value theory (MAVT) (Keeney and Raiffa 1976, Edwards 1977, Edwards and Barron 1994). Other procedures they have developed and applied include Roy's (1968, 1996) ELECTRE (election et choix traduisant la réalité), PROMETHEE (preference ranking organisation method for enrichment evaluations) by Brans et al. (1996), TOPSIS (technique for order preferences by similarity to ideal solutions) by Hwang and Yoon (1981), and Hwang et al. 1993), and the analytic hierarchy process (AHP) (Saaty 1977, 1980, 1982). People often refer to ELECTRE and PROMETHEE as representing the European School, and TOPSIS and the AHP (and others) as representing the American School. (Mollaghasemi and Pet-Edwards 1997 and Belton and Stewart 2002 describe most of these multicriteria decision-making (MCDM) methods.)

Since its development 25 years ago, the AHP has become a widely popular MCDM procedure in the US and in other countries. The AHP has gained wide acceptance among academics and practitioners (Golden et al. 1989, Forman and Gass 2001, Golden and Wasil 2003). It is the workhorse for solving multicriteria problems that have a finite number of alternatives to be put in rank order by priority (weight). The AHP seems to have replaced MAUT and MAVT for solving such real-world MCDM problems. As an exponent of the AHP, I introduced it to my MBA and undergraduate management science students in the early 1980s and was the first to discuss it in a text (Gass 1985). I am an AHP fan. But as the new kid on the block, the AHP has had to compete against established methods, in particular, MAUT. Many academics schooled in MAUT debated the efficacy of the AHP.

In the mid-1980s, MCDM researchers began to compare the AHP to MAUT. They found that the AHP does not adhere to the basic von Neumann and Morgenstern axiomatic structure of normative utility theory as incorporated in MAUT, and they raised other concerns. I tried to reconcile the criticisms of the AHP with my understanding of AHP and MAUT and joined E. Forman in describing our view of the debate (Forman and Gass 2001). We concluded:

> The AHP has been tested in the marketplace. Its acceptance as a new paradigm for decision analysis has been remarkable. . . . The AHP is theoretically sound, readily understood, easily implemented, and capable of producing results that agree with expectations (p. 485).

I thought we had laid the matter to rest. But Smith and von Winterfeldt (2004, p. 568) reviewed past *Management Science* papers in DA and made the following statement:

> While many in the decision analysis community (ourselves included) follow Dyer in believing the AHP to be fundamentally unsound, others (including Saaty, Harker, and Vargas) disagree and the AHP is still widely used in practice today.

That statement sends the wrong message to the OR community. That statement motivated me to write this column. (Smith and von Winterfeldt were joint editors of *Management Science*'s DA department in 2004.)

In discussing the AHP, Smith and von Winterfeldt (2004, p. 568) began as follows:

> One of the more acrimonious debates in *Management Science* has concerned the Analytic Hierarchy Process (AHP).

Management Science has published few papers on the AHP. For our discussion, the following five are of interest: Saaty (1986), Harker and Vargas (1987), Dyer (1990), Saaty (1990), and Harker and Vargas (1990). Saaty (1986) provided a full and detailed set of axioms that define the boundaries of the AHP. Harker and Vargas (1987) reviewed the theory of the AHP and answered criticisms of the methodology. Dyer (1990) reviewed the criticisms of the AHP, concluded that the AHP is "flawed," and offered a resolution of the difficulties by synthesizing the "flawed" AHP and MAUT. Finally, Saaty (1990) and Harker and Vargas (1990) rebuted Dyer's arguments. Why the debate?

The main criticisms of the AHP concern its measurement scale, rank reversal, and transitivity of preferences. In Saaty's original AHP paper (1977) and in his book (1980), he describes a measurement scale (termed the fundamental 1–9 scale) for measuring the importance of one criterion over another or the preference of one alternative with respect to another evaluated against a criterion. Rank reversal occurs when, after the decision maker (DM) ranks the given set of alternatives, the DM adds a new, irrelevant alternative to the set. Upon ranking the modified set, the DM finds that the previously highest ranked alternative is downgraded. Transitivity between preferences means that if alternative A is preferred to alternative B, and alternative B is preferred to alternative C, then alternative A is preferred to alternative C. I want to concentrate on rank reversal and transitivity, as these are the aspects Smith and von Winterfeldt (2004) highlighted. They stated:

> While Saaty (1986) provided an axiomatic foundation for the AHP, these axioms conflict with the axioms of expected utility theory and have met with resistance from decision analysts (p. 568).

Rank reversal in utility theory is related to the concept of irrelevant alternatives as given by the following axiom:

> Adding new acts (alternatives) to a decision problem under uncertainty, each of which is weakly dominated (preferred) by or is equivalent to some old act, has no effect on the optimality or non-optimality of an old act (Luce and Raiffa 1957, p. 288).

For example, when buying a car, you first consider and rank three different ones (A, B, C) and find that A has the highest rank. You add a fourth, say an exact copy of C, and for the new problem, a ranking of the four cars now causes B to have the highest rank. MAUT proponents use this type of problem in their attack on the AHP because rank reversal can occur under AHP but not under MAUT. (Why anyone would structure a decision problem in this manner is unclear; in defining decision analysis Keeney and Raiffa 1976 state that the decision problem starts with "*prespecified* alternatives.") Rank reversal can and does occur in many real-world decision problems. Luce and Raiffa (1957, p. 288) describe such a situation, but then they, and most DA researchers, conveniently rule out such possibilities because rank reversal confounds the axiomatic requirements of MAUT. The AHP has no comparable axiom. In the AHP, rank reversal, which is a function of how the decision problem is formulated, can occur for proper methodological reasons. Saaty (1987, 1990) puts the case to rest by describing how one can or cannot allow for rank reversal when using the AHP; Forman and Gass (2001) also discuss this matter.

The requirement for transitivity of preferences is another axiom of utility theory (Luce and Raiffa 1957, p. 25). But intransitive situations can occur when one attempts to resolve complex multicriteria problems, especially when one must evaluate many alternatives (Tversky 1969). Luce and Raiffa (1957, p. 25) noted:

> No matter how intransitivities arise, we must recognize that they exist, and we can take little comfort in the thought that they are an anathema to most of what constitutes theory in the behavioral sciences today.

Further, Fishburn (1991, p. 115) stated:

> Transitivity is obviously a great practical convenience and a nice thing to have for mathematical purposes, but long ago this author ceased to understand why it should be a cornerstone of normative decision theory.

Without the transitivity axiom, utility theory does not work. The AHP does not assume transitivity and, given an intransitive situation, the DM has available procedures for finding and mitigating it if deemed appropriate (Gass 1998).

The AHP is not an extension of MAUT. As Saaty (1990, p. 259) noted:

> From its axioms to its procedures, the AHP has turned out to be historically and theoretically a different and independent theory of decision making from utility theory.

In this debate, the questions to be addressed are the following: Because the AHP is not an extension of MAUT, why is it criticized for not adhering to axioms of MAUT? Must all procedures for resolving multiattribute problems follow the rules of MAUT? Are the axioms of MAUT sacrosanct?

I have found it strange that MAUT researchers tend to ignore criticisms about the viability of MAUT's axiomatic base and related applied issues, while rejecting the AHP for not adhering to MAUT's view of the world. Criticisms of MAUT, extending over many years, include Allais' (1953) paradox and related studies that show "that subjects systematically make choices that violate properties required by expected utility" (Keller 1992, p. 4); Ellsberg's (1961) urn paradox, Kahneman and Tversky's (1979) work, and McCord and deNeufville's (1983, p. 301) statement:

> The conclusion is that the justification of the practical use of expected utility decision analysis as it is known today is weak.

Many papers in the MAUT literature discuss these criticisms; most of their authors either ignored their implications or offered ways to mitigate them by stretching the given axiomatic boundaries. Howard (1992) did not feel that any such stretching is necessary. He said that DA is a normative method that "should govern decision making" (p. 51). That is, the normative axioms described by von Neumann and Morgenstern rule. For DA problems, he saw deviations from the norms as:

> . . . approximations that are appropriate when applying the norms in practice. These approximations are not mistakes in the sense that they are violations of the norms of decision making, but are rather the interpretations required to apply the norms sensibly in the world (pp. 51–52).

I interpret this to mean that, within the DA world, it is all right to apply methodological adjustments ("approximations") to utility theory's axiomatic normative base if they help to resolve a particular decision problem in a manner acceptable and justifiable to the DM. I have no argument with this view. Such adjustments are part of all modeling-based analyses, a case in point being a mathematical-programming problem in which nonlinear conditions are approximated by linear constraints.

I wonder why Howard's (1992) approach to decision modeling does not extend to the AHP and its use in practice. I also wonder why the DA critics do not accept Saaty's (1990) and Harker and Vargas's (1987, 1990) discussions in their resolution of the criticisms raised against the use of the AHP. I do not use EUT or MAUT, but I respect the practice of those who do in their astute analyses that mitigate the criticisms raised against EUT and MAUT (Keeney 1992, pp. 57–72). I agree with Keeney (p. 58) that "decision making is concerned with helping people make informed, and hopefully better, decisions." This is the aim of both a MAUT analysis and an AHP analysis.

There have been many hundreds (if not thousands) of successful applications of the AHP. Forman and Gass (2001) described 26 applications of the AHP for problems in choice, prioritization and evaluation, resource allocation, benchmarking, quality management, public policy, health care, and strategic planning by such organizations as Xerox, IBM, the US Department of Energy, the US Department of Defense, NASA's Johnson Space Center, Rockwell International, Air Products and Chemicals, and the Rochester General Hospital. The American Society for Testing and Materials has included the AHP in

its publication for performing multiattribute decision analysis to evaluate buildings and building systems (ASTM 1998). The AHP-based software Expert Choice has been employed in over 50 countries (Forman and Selly 2001). The site www.expertchoice.com lists many other organizations that have used the AHP and provides capsule summaries of some of their applications; Zahedi (1986), Golden et al. (1989), and Saaty and Vargas (2000) describe successful applications. The September 2003 issue of *Computers and Operations Research* is dedicated to "Celebrating 25 years of AHP-based decision making." The issue's editors (B. Golden and E. Wasil) noted:

> Operations research practitioners around the world have repeatedly embraced AHP as a methodology that can produce insightful results to difficult, real-world decision problems (p. 1419).

International symposia on the AHP have been held in Pittsburgh, Washington, Tianjin, Vancouver, Kobe, Berne, Bali, and Hawaii (July 2005). Forman and Selly (2001) and Bodin and Gass (2003) discuss the pedagogical aspects of the AHP.

To sum up, I urge the OR/MS community not to reject the use of the AHP because of Smith and von Winterfeldt's (2004) comments. The arguments they posed to bolster their claim that "the AHP is fundamentally unsound" (p. 485) have been refuted in theoretical, methodological, and practical terms. Further, those unfamiliar with the AHP should read the original papers that established the field and the responses to AHP critics. AHP researchers and practitioners should consider the full range of INFORMS journals as suitable outlets for their work. I hope that the editors of INFORMS journals, especially those responsible for the decision analysis departments, encourage submission of papers on the AHP and that they become more receptive to them.

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