Chapter 8

Resource Allocation

What is resource allocation all about?

According to Churchman[1], management is responsible for allocating resources in order to achieve an organization's purpose.

> "In organizations, the decision-making function is the responsibility of management. In order to execute its responsibility, an organization's management requires information about the resources available to it and their relative effectiveness for achieving the organization's purpose. Resources are acquired, allocated, motivated and manipulated under the manager's control. They include people, materials, plant and equipment, money, and information.[2]"

The notion of an "organization's purpose" may be somewhat vague, especially if the organization is large. In such cases, the purpose is sometimes stated as its vision, or goal. The vision or goal is attained through the achievement of multiple (often numerous, and competing) objectives. We have defined *rationality* as the achievement of objectives—in this context, the objectives that will best fulfill the organization's purpose. An important question then is, how does an organization rationally allocate its resources in order to achieve its goals?

We have often asked executives how resources are allocated in their organization. After an initial blank stare, we hear responses like — "The executive committee meets and....", or "We use the budget contained in the five year plan....", or "We start with last year's resource allocation and make adjustments based on....". These responses beg the question of *how* resource decisions are made in some 'rational' way that is based on their relative effectiveness for achieving the organization's purpose(s) or objectives.

In order to make resource decisions in such a rational way, an organization must do the following:

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[1] "Measurement for Management Decision: A Perspective", Richard O. Mason and E. Burton Swanson, reprinted from the *California Management Review*, Vol 21, No 3 (Spring 1979), p14

[2] Ibid.

Identify / design alternatives (e.g., alternative R&D projects, or operational plans for alternative levels of funding for each of the organization's departments)

Identify and structure the organization's goals into objectives, sub-objectives, sub-sub-objectives, and so on.

Measure how well each alternative contributes to each of the lowest level sub-objectives

Find the best combination of alternatives, subject to environmental and organizational constraints.

The Silverlake Project

Before looking at these activities in detail, a look at the underlying conditions at one organization that was able to successfully make 'rational' resource decisions will help people in your organization relate to the difficulties of traditional approaches and, hopefully, welcome some improvement in your current resource allocation process.

In the forward to *The Silverlake Project*: Transformation at IBM[3], Tom Peters wrote:

> "This is a remarkable tale. IBM had an amazing success with its AS/400 mid-range family of computers. ... Disarray is too kind a word for IBM's position in the growing, important mid-range computer business in 1986. Competitors were attacking from every point on the compass. A major project aimed at righting the ship was an expensive fiasco, and had to be canceled. ... Twenty-eight months later, a relatively neglected development lab in Rochester, Minnesota was the talk of IBM. Two years later, the same group had added the prestigious Baldridge quality prize to impressive gains in market share and profitability. The enormity of the shift is hard to overstate. (Tally the revenue the group generates and you'd have the world's second largest computer company - behind IBM, of course.)"

The process was not an easy one however, as evidenced from some of the participant's observations:

> "The skirmishes started from day one."

> "What made it even more frustrating was that everyone seemed to have a legitimate claim for funding ahead of someone else. But in reality, there just wasn't enough money to fulfill everyone's wish list."

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[3] Bauer, Roy, A., Collar, Emilio, and Tang, Victor *The Silverlake Project*: Transformation at IBM Silverlake Project: Transformation at IBM. Oxford University Press, New York, 1992.

"Our quandary was a quintessential one. Like so many enterprises, we had to cope with the untenable demands of satisfying virtually unlimited needs on a very limited budget."

"The real problem, though, was that each organization was evaluating and committing to their piece-parts with no real understanding of how their decisions were affecting the whole. And although we had a consolidation process for the piece-parts, we were unable to convincingly demonstrate that the final result would balance the needs for market share, technology leadership, and for being affordable and competitive. We had no methodical, objective basis for making these tradeoffs, especially in a way that served our overall business objectives. Consequently, we had trouble explaining our decisions to those affected or to those we answered to in a credible, defensible way. Indeed, for as rational as most managers want to be, when it comes to allocating resources, they frequently lack the methodologies for making systematic decisions. So they're forced to act by decree. Or whim. Or, worse, they wind up, like a practiced old Capitol Hill pol, making stopgap attempts at appeasing the sundry, and often competing, interests found in most organizations."

"So one of two things happened. We wound up taking funds from everyone, right across the board. Or we'd simply cancel a part of the Systems Plan. ... But no mater what form it took, this give and take ... caused no small amount of consternation, especially for those of us on the giving end."

"Victor Tang, who was in charge of planning, and Emilio Collar, who oversaw market analysis, watched as Furey, Schwartz, and other general managers struggled with such decisions. They figured there had to be a better way. They viewed this struggle as an issue fundamental to strategic decision-making, one only compounded by the vast complexity of global markets. It simply begged for a more rigorous and systematic process. So together they embarked on an approach for setting priorities as the basis for allocating resources... To their way of thinking, the only way to make sound decisions for allocating resources was to create a priority ranking for each and every one of the line items themselves."

"But what would be the basis for such a ranking? Markets? Technological considerations? Financial objectives? It actually had to be done on the basis of all three considerations. Not only that, we would have to take all three into account in a *balanced* way - one that would accomplish the broadest goals as well as the narrower business objectives..."

Does any of this sound at all familiar? What do you do at your organization? Here is what they did at IBM Rochester:

"Thankfully, Collar found a model for helping us make our priority-setting decisions - a methodology to render the ranking process more objective and systematic. It also allowed us to take any number of criteria into consideration. In short, it enabled us to deal with our situation in all of its complexity. It was called the Analytic Hierarchy Process (AHP),

"...as part of AHP, you, the decision maker, get to build the hierarchy. You establish the goal, the criteria, and the options. In so doing, you can actually bring ideas, anecdotal experience, even emotion into the process."

"Automated in the form of a relatively inexpensive software program called "Expert Choice", AHP is an extraordinarily powerful decision-making tool. It brings structure to a decision-making, yet it's flexible because you get to design a hierarchy of goals, criteria, and options customized to the particular problem at hand. It can be used with groups, and, as a collaborative effort, it can bring consensus to decision-making. It allows you to quantify judgments, even subjective ones. *It also forces you to consider the interdependencies of your criteria to meet your goals.* AHP pinpoints for you where the impacts are the highest or inconsistent. You can play "what-if" games with it..."

"AHP allows you to set priorities by taking several factors into consideration—factors that interplay and affect each other. In building the hierarchy, you can have goals and sub-goals. You can have several layers of criteria. You can also deal with several layers of options. In other words, you can address extraordinarily complex situations, ones with multi-dimensions that have interconnections every which way."

"AHP became the template we imposed on our efforts to rank the line items of the System Plan so that we could earmark funds in a much more methodical, rationally defensible way."

"All this had very real consequences. In the past, we would sometimes chase markets simply because some highly placed executive decreed we should. Usually these decrees were based on an anecdotal experience with a particular customer or industry. But with our priority ranking in hand, it became easier to fend off such unjustified dictates. ... For the first time, we could confidently articulate what businesses we were in and, more importantly, which ones we were not."

"We had come up with a process for setting priorities and, thus, for making decisions on earmarking resources. It's a rare organization that doesn't find itself in need of some similarly systematic process. In today's competitive global marketplace, where almost everyone is finding themselves having to do more with less, figuring how much money should be spent where may be one of a manager's most difficult tasks. But the problems and pain can be obviated by setting priorities. Its simple: Before you decide how to budget, you've simply got to know what's most important, especially in reaching your organization's overarching goals in a holistic, balanced way. As straightforward as that sounds, however, it's hardly a simple task. Making decisions today depends on taking into consideration any number of interdependent goals, criteria, and options. As we proved though, relying on a model can make the job of setting priorities to allocate resources much more methodical and objective and, therefore, much more credible and defensible."

The details of how they did it are not presented in *The Silverlake Project*, although the authors do discuss some specifics of what they did. We will next present the details of several approaches to allocating resources that can be used to fit in with any organization's objectives and constraints. Before looking at the details, it is important to once more focus on the big picture—what needs to be done when allocating resources and what the consequences of a rational, systematic resource allocation methodology can be. Bauer, Collar, Emilio, and Tang, authors of *The Silverlake Project, write:*

"Making tradeoffs is a fact of organizational life, especially in an era of doing more with less. So priorities have to be set. But those priorities must be determined on the basis of the enterprise's overall objectives. Resource decisions need to be made holistically, that is, with their consequences to the entire enterprise and all its parts in mind."

"Setting priorities - priorities that will serve as a guide to resource decisions—shouldn't be a matter of guesswork. It must be done through a process that's as systematic as possible. And one which produces repeatable results. This is precisely what we did—not only in allocating resources, but ultimately, in determining the shape of the entire Silverlake Project."

Methodology Overview

There are a variety of ways to achieve a systematic, rational, and defensible allocation of resources that will provide a competitive advantage to an organization. The methodology discussed below is quite flexible and can be adapted to a wide variety of situations and constraints. As outlined above, the methodology consists of the following steps[4]:

1. Identify/design alternatives

2. Identify and structure the organization's goals and objectives

[4] Keep in mind that these 'steps', are part of a 'process', in which iteration is extremely important.

3. Prioritize the objectives and sub-objectives
4. Measure each alternative's contribution to each of the lowest level sub-objectives
5. Find the best combination of alternatives, subject to environmental and organizational constraints

Identify/design alternatives

Expertise in the art and science of identifying and/or designing alternatives lies in the domain of the decision makers, who have many years of study and experience to bear on this task. Our goal here is not to tell them how to do this, but, instead, to help them better measure and synthesize in order to better capitalize on their knowledge and experience.

Even if alternatives have already have been identified, e.g. R&D project proposals as responses to a request for proposal (RFP), it might be possible to augment or redesign these as part of the 'process'. For example, if, after one 'iteration' of the resource allocation methodology for allocating funds to internal R&D project proposals, it may be to the organization's benefit to make known the 'preliminary' allocation as well as the details of the evaluation so that proposors can revise their proposals, and, in the process, improve their contribution to the organization's objectives. Of course, there are rules that must be employed in this context to insure fairness. For example, government agencies may, by law or regulation, have to limit the process to one iteration with no opportunities for the proposors to 'improve' their proposals. Such laws and regulations sacrifice 'quality' for 'fairness'. While rules, laws or regulations that limit feedback and iteration are intended to make the process 'fair', the tradeoff between 'fairness' and 'quality' of results should be carefully considered when deciding on the 'rules' for the resource allocation process.[5]

Instead of deciding which alternatives to fund and not to fund, as in the case of R&D project selection, a more common resource allocation activity is the periodic allocation of an organization's basic budget. Here, the alternatives are not which departments to fund, but instead, at what level

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[5] The choice of procurement rules is itself a multi-objective decision that should be addressed before the actual procurement process begins.

should each department be funded. Each department, or organizational unit, can design their operation at alternative levels of funding, e.g., equal to last years funding, 10% above last years funding level, 10% below last years funding level, 20% above, etc. If an organization faces a budget cut, say 10%, for example, cutting each organizational unit by 10% (sometimes called across the board cuts) may seem fair, but it is not competitive. It stands to reason that in today's fast changing world, in order to be competitive, some units should be cut perhaps 50%, 80% or entirely, in order to increase the budgets of other units by significant amounts. The resource allocation methodology presented below makes this a practical approach.

Identify and structure the organization's goals and objectives

The main message of this book is that decisions must be made on the basis of achievement of objectives. And so it is with resource allocation decisions. As Bauer, Collar, Emilio, and Tang advise in The Silverlake Project, "priorities must be determined on the basis of the enterprise's overall objectives. Resource decisions need to be made holistically, that is, with their consequences to the entire enterprise and all its parts in mind." And so the entire enterprise's goals and objectives must be addressed. This may not seem easy to do in a large enterprise or organization; however, it can be done the same way that large organizations are typically organized, that is, hierarchically.

Most organizations already have a statement of goals and objectives (sometimes organized as values, goals, and objectives). A good bet is that they are already structured hierarchically. Where are they kept? In some organizations they might be framed and hung on the walls to help remind employees and inform customers of the organization's values, goals and objectives. In other organizations they might be in 'the blue book' or 'the black book' that is referred to from time to time. It is important to study these goals and objectives to see if they are 'living' or long forgotten soon after they were drafted. If they are still living, and up to date—taking into account the fast changes in business and society, (locally, nationally, and globally), they will serve as the driving force behind the allocation of resources. Otherwise, they must be revised, brought back to life, and continually examined to keep them current. Once people realize that the enterprise's overall objectives will serve as the basis for the allocation of resources, there will be a keen interest in keeping them current and relevant!

The hierarchy of objectives, sub-objectives, sub-sub-objectives, and so on, must be broad enough to encompass every existing or desired activity that is part of the resource allocation process. If not, proponents of an activity will not be able to show where and how much the activity can contribute to the organization's objectives. Once more, we need to be open to iteration since it might not become obvious that specific objectives were overlooked until the activities are rated in a subsequent step of the process.

Top level executives understand and can best make judgments about the relative importance of the main organizational objectives, and possibly the sub-objectives. They know very little about the detailed alternatives vying for funding. Lower level management and operational personnel can best make judgments about the relative importance of the lowest level sub-objectives and about how much contribution each alternative contributes to the lowest level sub-objectives. They often don't appreciate or understand top management's strategic direction or change in direction. The systematic approach presented below makes it possible to synthesize knowledge, experience, and insights across many levels within a large organization, something that has not been possible to do in the past. Without such a synthesis, it is virtually impossible to achieve a rational, competitive allocation of resources.

Prioritize the objectives and sub-objectives

The relative importance of the objectives and sub-objectives must be established in order to make a rational allocation of resources. Neglecting to do so is a mistake. Assuming that all the main objectives are equally important is a mistake. The pairwise comparison process and team methods discussed in this text provide a straightforward, informative, and reliable way to prioritize the organization's objectives. Remember, this is a 'process'. After establishing priorities for the organization's objectives, and subsequently deriving a 'preliminary' allocation based on these priorities, the priorities and judgments that served to derive the priorities should be re-examined and revised as necessary. The prioritization of the organization's objectives during the resource allocation process leads to another important benefit. In top management's quest for excellence and response to shifts in direction brought about by changes in the environment and competitive forces, what better way is there to convey their priorities to the organization at large? If these priorities are not conveyed, it is almost inevitable that individuals or departments which formerly provided valuable services to an organization, will, because of ignorance of changes in the organization's primary objectives or their relative importance, someday be surprised to find their contributions are no longer of value.

Measure Alternatives' Contributions

Having prioritized the organization's objectives and sub-objectives, the next step is to evaluate how much each proposed activity (or each possible level of funding for each activity) would contribute to each of the lowest level sub-objectives. This could be done by a pairwise comparison process, but because there will normally be many, possibly hundreds or thousands of activities or levels of funding, the ratings approach is customarily used.

Find the Best Combination of Alternatives

After prioritizing the organization's objectives and sub-objectives, and rating the contribution of the competing activities to the lowest level objectives, we have ratio scale measures of the relative contribution of each alternative to the overall objectives of the organization. We claim that there is no way to rationally allocate resources without such measures and suggest that the derivation of such measures for a large, diverse organization without the approach detailed here is almost impossible to accomplish! We will next consider two different basic situations in which we seek to find the best combination of alternatives for the allocation of resources. In the first situation, which we will refer to as Discrete Alternative Resource Allocation, each project/activity is a separate or discrete unit. (There may be a variety of dependencies between these units.) For example, we might be allocating resources to discrete proposals in response to an RFP. In the second situation, which we will refer to as Activity Level Resource Allocation, each project or activity is represented at one or more levels of funding but an additional constraint is that each project can be funded at only one level. (Here too there may be a variety of dependencies between projects,).[6] For example, each department of an organization may design several alternative levels of funding; the resource allocation methodology is to find the levels for each department that produces the best overall results for the organization. For each of these two situations we will look at two different approaches, one that focuses on the maximization of benefit/cost and the other that focuses on the maximization of benefit.

Discrete Alternative Resource Allocation

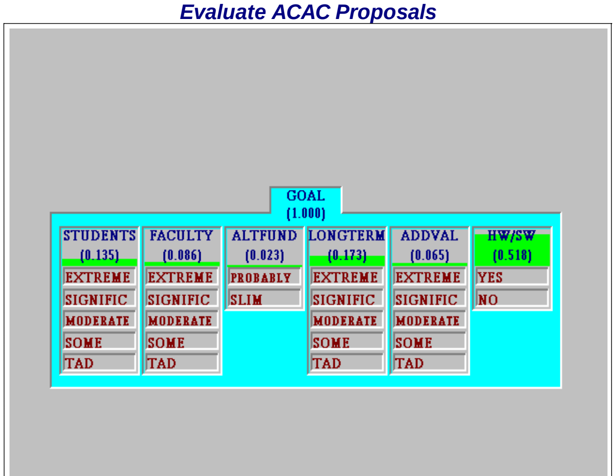
George Washington University Academic Computing Advisory Committee

We will illustrate the allocation of resources to alternative projects/activities with a rather small, but typical example. The Academic Computing Advisory Committee of the George Washington University had $15,000 available to fund proposals relating to the use of computers. After publishing a short RFP, eleven proposals were received. The total amount requested for all the proposals was $34,430. The committee, consisting of nine faculty members had to decide which of the proposed projects to fund. Can you imagine getting nine faculty members to agree?

After discussing 'criteria' or, what we prefer to call 'objectives', the following Expert Choice model was developed:

[6] After looking at these two situations, it will be easier to understand the general situation which is simply a hybrid of the two.

Evaluate ACAC Proposals



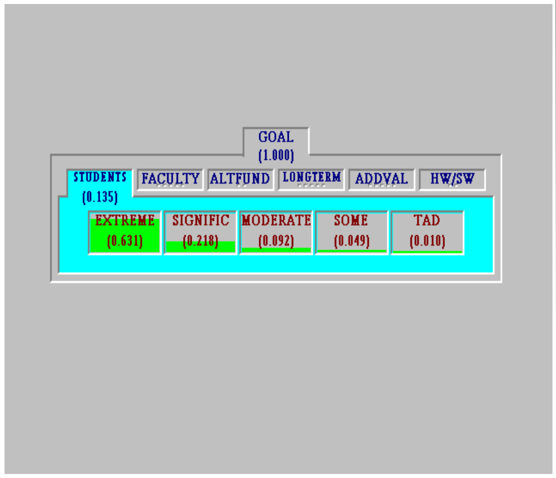
Abbreviation	Definition
ADDVAL	adding value to existing resources
ALTFUND	Availability of alternative sources
EXTREME	
FACULTY	Contribution to Faculty
HW/SW	Hardware / Software
LONGTERM	Long term benefit
MODERATE	
NO	
PROBABLY	
SIGNIFIC	
SLIM	
SOME	
STUDENTS	Contribution to students
TAD	
YES	

(Priorities shown are 'Local' -- relative to parent node.)

Figure 1 – Decision Hierarchy

Through a process of discussion and pairwise comparisons (the committee used keypads to enter individual judgments, which were then aggregated) the priorities of the objectives (shown in Figure 1) were derived. (A complete discussion of group or team decision-making begins on page 115). Since the committee's focus was on computer hardware and software, and since some of the proposals were only peripherally related to hardware and software the HW/SW objective was judged to be the most important. Expected long-term benefits to the University were second most important, followed by contribution to student and faculty activities (learning, research etc.) The objective of adding value to existing resources was next in importance, followed by the objective of funding projects for which there was little likelihood of alternative funding sources. Note that the priorities possess the ratio level of measurement. The priority of the HW/SW objective is 2.99 times the priority of the second most important objective, long-term benefits to the University. Had an ordinal scale of six items been used, the most important objective would have a value of 6, the next most important objective a value of 5, and the ratio would have been only 6/5 or about 1.2.

Evaluate ACAC Proposals



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SIGNIFIC	
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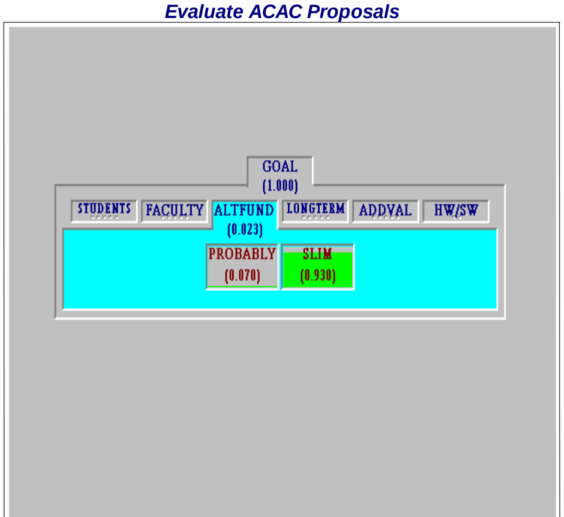
STUDENTS	Contribution to students
TAD	

(Priorities shown are 'Local' -- relative to parent node.)

Figure 2 – Intensities

The intensities below each of the objectives were also prioritized with pairwise comparisons. For example, a project that is judged to make an extreme contribution to students will receive a priority for that contribution of about 63 times that of a project that makes only a 'tad' of a contribution to students, as can be seen in Figure 2. This is quite different than an ordinal scale where the ratio would be 5 to 1 instead. The intensities and their scale can be different under each of the objectives. The intensities for the alternative funding sources is shown in Figure 4.

Evaluate ACAC Proposals



Abbreviation	Definition
ADDVAL	adding value to existing resources
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LONGTERM	Long term benefit
PROBABLY	
SLIM	
STUDENTS	Contribution to students

(Priorities shown are 'Local' -- relative to parent node.)

Figure 3 – Alternative Funding

After the objectives and intensities are prioritized, each of the alternative projects is rated with respect to each of the lowest level objectives, as shown in Figure 4.

Alternatives	TOTAL	COSTS	STUDENTS .1350	FACULTY .0859	ALTFUND .0225	LONGTERM .1730	ADDVAL .0654	HW/SW .5182
Proposal One	0.684	2,000	0.417	0.241	0.384	SIGNIFIC	0.320	YES
Proposal Two	0.598	1,622	0.202	0.146	0.422	0.116	0.160	YES
Proposal Three	0.742	1,515	0.782	SIGNIFIC	PROBABLY	0.395	0.285	YES
Proposal Four	0.762	1,280	0.709	SIGNIFIC	PROBABLY	0.490	0.490	YES
Proposal Five	0.090	7,500	0.055	0.121	PROBABLY	0.295	0.212	NO
Proposal Six	0.079	4,375	0.134	0.136	PROBABLY	0.202	0.117	NO
Proposal Seven	0.722	3,187	0.672	SIGNIFIC	PROBABLY	SIGNIFIC	SIGNIFIC	YES
Proposal Eight	0.575	565	0.490	0.256	PROBABLY	0.329	0.373	0.780
Proposal Nine	0.647	4,000	0.220	0.220	0.868	0.220	0.345	YES
Proposal Ten	0.709	3,300	0.380	0.256	PROBABLY	0.468	0.541	YES
Proposal Eleven	0.647	5,086	0.395	0.167	0.589	0.212	0.175	YES

Figure 4 – Combined Ratings

The entries in the columns representing the objectives (STUDENTS, FACULTY, etc.,) are either one of the intensities for that column (for which we derived is a ratio scale value) if all nine faculty members agreed on the rating, or a decimal value which represents the average of the intensities of the nine faculty members. The values in the Total column are ratio scale measures of the contribution each project is expected to make to the organization's objectives, or, as Bauer et. al assert:

> "Before you decide how to budget, you've simply got to know what's most important, especially in reaching your organization's overarching goals in a holistic, balanced way."

In this simple example, there are only six 'overarching goals' or objectives, each represented by a column in the matrix. In a typical large organization there may be hundreds of columns, structured hierarchically.

What does the total value .684 for Proposal One in Figure 4 mean? By itself nothing, but it means a great deal when compared to the total values for the other alternatives—Proposal One contributes about 8.7 times as much as Proposal Six, for example. Because the totals are ratio scale numbers, we can normalize without changing the ratios. The totals in Figure 4 are normalized such that an 'ideal' alternative, one that rated best in every column would have a value of 1.0. Figure 5 shows the two other normalizations: one normalized so that the best alternative is 100%, and another normalized so that the priorities add to 1.0. Any of these three

Alternatives	% OF MAX	PRIORITY
Proposal One	89.813	0.109
Proposal Two	78.493	0.096
Proposal Three	97.377	0.119
Proposal Four	100.000	0.122
Proposal Five	11.747	0.014
Proposal Six	10.397	0.013
Proposal Seven	94.798	0.115
Proposal Eight	75.505	0.092
Proposal Nine	84.901	0.103
Proposal Ten	93.106	0.113
Proposal Eleven	84.942	0.103

Figure 5 – Percent of Maximum

normalizations of the projects' expected[7] total benefit could be used in determining the best allocation of resources. How the numbers are used depends on the circumstances and constraints of the situation. If, for example, each project could be partially funded and would produce benefits in proportion to the amount funded, we might decide to allocate the $15,000 to the projects on the basis of the normalized priority column, so that Project One would get 10.9%, Project Six 1.3% and so on. However, the assumptions here are that each project requires a given amount of funding (expressed as Costs in Figure 4) in order to deliver the benefits in the total column—that is, we can not fund a project at some fraction of its stated cost[8]. How then should we decide which projects to fund? We will look at two approaches, depending on whether we are interested in achieving the highest benefit/cost ratio or the highest benefit.

[7] We refer to the benefits as 'expected' because the ratings were made on the basis of what the projects would be expected to contribute. A more formal 'expected' contribution could be derived by including scenarios between the goal and top level objectives in the AHP model and making judgments about the relative likelihood of each of the scenarios.

[8] Mechanically, it is just as easy, perhaps even easier, to perform the resource allocation allowing for partial funding of projects.

	Alternatives	Benefit	Cost	B/C	Cum C.	Cum B.
Alt 008	Proposal Eight	0.092	565	162.832	565	0.092
Alt 004	Proposal Four	0.122	1280	95.313	1845	0.214
Alt 003	Proposal Three	0.119	1515	78.548	3360	0.333
Alt 002	Proposal Two	0.096	1622	59.186	4982	0.429
Alt 001	Proposal One	0.109	2000	54.500	6982	0.538
Alt 007	Proposal Seven	0.115	3187	36.084	10169	0.653
Alt 010	Proposal Ten	0.113	3300	34.242	13469	0.766
Alt 009	Proposal Nine	0.103	4000	25.750	17469	0.869
Alt 011	Proposal Eleven	0.103	5086	20.252	22555	0.972
Alt 006	Proposal Six	0.013	4375	2.971	26930	0.985
Alt 005	Proposal Five	0.014	7500	1.867	34430	0.999

Figure 6 – Benefit Cost Ratios

Benefit/Cost Ratios—Sort and Allocate

Given the benefits and the costs, we can easily calculate the benefit/cost ratios, and sort from high to low as shown in Figure 6.[9] We can then allocate funds to the projects starting with the project with the highest benefit/cost ratio and continuing until the $15,000 budget is used up. This will produce the largest benefit/cost ratio while funding as many projects as possible within the budget constraint. Looking at the cumulative cost column, we see that if we were to do this, we would fund Projects 8, 4, 3, 2, 1, 7 and 10. Projects 9, 11, 6 and 5 would not be funded.

Figure 7 contains a benefit/cost efficient frontier graph produced with Team Expert Choice. Notice that the curve is concave, signifying diminishing marginal benefit as additional projects are selected with lower benefit/cost ratios.

[9] The b/c ratios have been multiplied by 10^6.

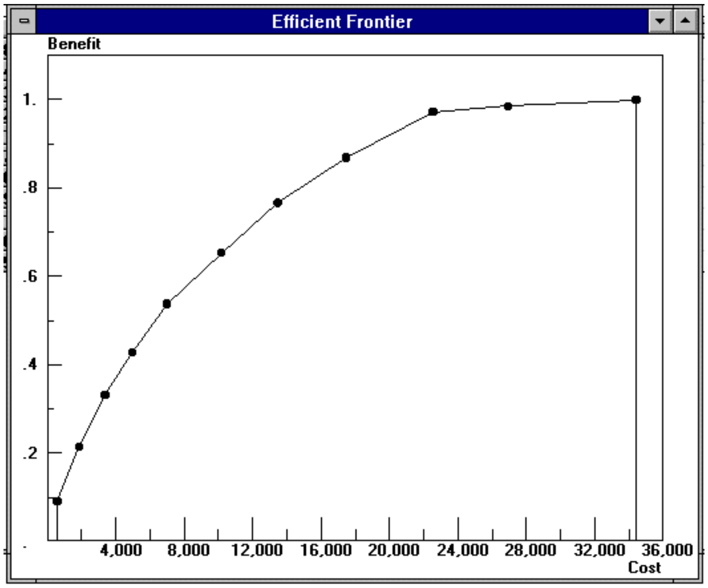


Figure 7 – Efficient Frontier

Before adopting this methodology, we should see when it makes sense to seek the highest cumulative benefit/cost ratio as our objective and when it doesn't. If we assume that an organization will consistently follow the philosophy of maximizing its cumulative benefit/cost ratio, then over time, they will also maximize their benefits[10]. This statement also assumes that there will be other comparable opportunities for additional resource allocations in the future, so whatever portion of the budget that is not expended will provide benefits commensurate with those activities currently being funded. However, these assumptions are not always valid and it is dangerous to take the maximization of benefit/cost ratios for granted. An example illustrating the danger of using a benefit/cost maximization will be presented after looking at the benefit optimization methodology.

[10] As compared to any other allocation that produces a lower benefit/cost ratio and, in the long run, allocates approximately the same total amount.

Maximizing Benefits -- Optimization

Another method for allocating resources seeks to find that combination of projects or activities that maximizes the total benefits without exceeding the given budget.[11] This problem can be formulated as a zero-one integer mathematical programming problem, sometimes referred to as a knapsack problem. Most spreadsheet programs today contain algorithms for solving such problems.[12] We will illustrate the solution to the preceding resource allocation exercise using the Solver tool in Microsoft's Excel. Mathematically, the optimization problem formulation is as follows:

Maximize $.109X_1 + .096X_2 + .119X_3 + ... + .103X_{11}$ (Benefits)

subject to: $2000X_1 + 1622X_2 + 1515X_3 + ... + 5000X_{11} <= 15000$ (Cost)

where: $X_1, X_2, X_3, ... X_{11} >= 0$

and $X_1, X_2, X_3, ... X_{11} <= 1$
and $X_1, X_2, X_3, ... X_{11}$ are integer.

The following steps will setup and solve the problem with Expert Choice and Microsoft's Excel:

I-a) From Expert Choice's ratings module, select the first three columns: (Alternatives, Priorities, Costs).

Then do an Edit Copy.

I-b) Start Excel and do an Edit Paste to the Excel spreadsheet.

I-c) Add a column with the heading DV's for the decision variables. The decision variables will be adjusted by the algorithm to be either 0, if a project is not to be funded, or a 1 if the project is to be funded. Put the value of 1 in each element of this column (for illustrative purposes only).

I-d) Add another column with the heading F.Benefits (for funded benefits). Enter a formula in the row corresponding to the first alternative in this column that multiplies the decision variable cell by the priority cell

[11] Compared to the ambiguity of what maximizing the cumulative benefit/cost ratio really accomplishes , the objective of maximizing benefits is straightforward, understandable, and doesn't depend on questionable underlying assumptions.

[12] Problems involving hundreds of alternatives might require special purpose optimization programs.

for this alternative. If, for example, the decision variable is in cell D3 and the priority in cell B3, the formula would be:

=D3*B3

I-e) Copy this formula down to the other cells in the F.Benefits column.

I-f) Add another column with the heading F.Costs (for funded Costs). Enter a formula in the row corresponding to the first alternative in this column that multiplies the decision variable cell by the cost cell for the first alternative. If, for example, the decision variable is in cell D3 and the cost in cell C3, the formula would be:

=D3*C3

I-g) Copy this formula down to the other cells in the F.Costs column.

II-a) Select the cell below the last alternative row in the F.Benefits column and click the Summation symbol on the Excel toolbar, then press Enter. Repeat this step for the F. Costs column.

II-b) Select the range of cells in the Decision Variable Column (all should be 1 right now). Do an Insert Name Define and enter DVS (for Decision Variables).

II-c) Move to the Total F.Benefits Cell.

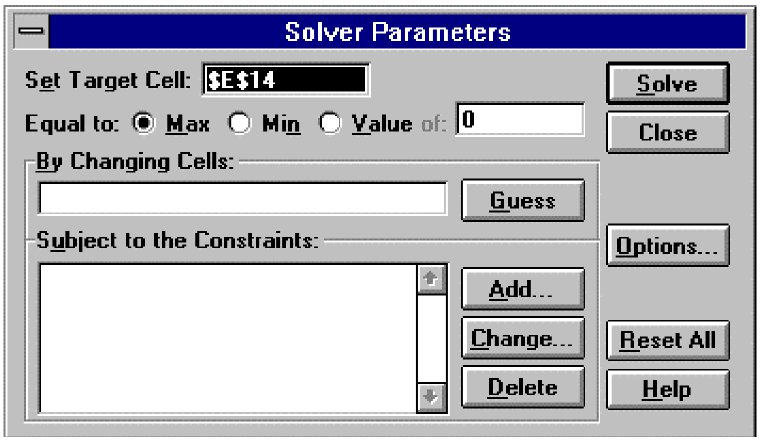
The spreadsheet should look something like Figure 8.

	A	B	C	D	E	F
1	Alternatives	PRIORITY	COSTS	DV's	F.Benefit	F.Cost
2						
3	Proposal One	0.109	2,000	1	0.109	2000
4	Proposal Two	0.096	1,622	1	0.096	1622
5	Proposal Three	0.119	1,515	1	0.119	1515
6	Proposal Four	0.122	1,280	1	0.122	1280
7	Proposal Five	0.014	7,500	1	0.014	7500
8	Proposal Six	0.013	4,375	1	0.013	4375
9	Proposal Seven	0.115	3,187	1	0.115	3187
10	Proposal Eight	0.092	565	1	0.092	565
11	Proposal Nine	0.103	4,000	1	0.103	4000
12	Proposal Ten	0.113	3,300	1	0.113	3300
13	Proposal Eleven	0.103	5,086	1	0.103	5086
14				Totals:	0.999	34430

Figure 8 – Funding All Alternatives

If every project were funded, the Total Funded Benefit would be about 1 (shown as .999) and the Total Funded Cost would be $34,430. This is obviously infeasible.

II-d) Click on the Tools menu.



If Solver is not listed as a choice, select AddIns and select Solver-Add In.

Figure 9 – Specifying Solver Parameters

Select Solver under the Tools menu. A Solver Parameters dialog box (see Figure 9) should appear.

Parameters must be set for three things: The Target Cell, The Decision Variables (By Changing Cells), and the Constraints.

III-a) The target cell is the cell we wish to maximize, in this case cell E-14, the total funded benefit. Since we pre-positioned to this cell, E-14 should already be selected. The Max button should also be selected by default.

III-b) From the Solver Parameter dialog box, type DVS (the name for the decision variables range) in the By Changing Cells: box.

III-c) The constraints are added next, as follows:

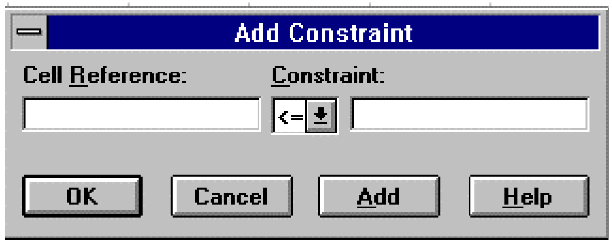


Figure 10 – Constraint Dialog Box

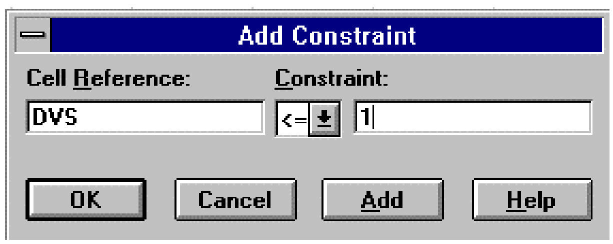


Figure 11 – Decision Variables <=1

Click the Add button and the Add Constraint dialog box (Figure 10) will appear:

1) Type DVS in the left box (Cell Reference); select <= in the middle box; and type 1 in the right box.
Then click Add to add another constraint.

2) Type DVS in the left box; select >= in the middle box; and type 0 in the right box (see Figure 11). Then click Add to add a third constraint.

3) Type DVS in the left box; select INT in the middle box; and then click Add to add a fourth constraint. (Note: Later versions of Solver have a 'Binary' option which can be used instead of steps 1-3 to specify that the decision variables must be integer values of 0 or 1).

4) With the left box active, click on the Total F.Costs in the Spreadsheet; select <= in the middle box; and type 15000 in the right box.[13]

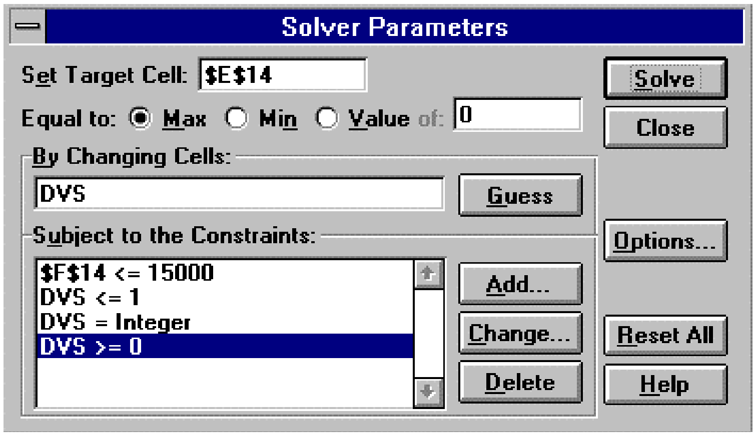


Figure 12 – Specified Parameters

Then press OK. The Parameters should look like Figure 12.

IV-a) Click on Options, select Assume Linear, then OK.

IV-b) Click on Solve. (If a message box appears stating the maximum iteration limit was reached, select Continue). When the Solve Results box appears, click the OK button.

The Spreadsheet should look like the Figure 13.

[13] Instead of entering $15000 as a constant, it can be entered into a cell which is referenced in right hand side of the constraint dialog box.

	A	B	C	D	E	F
1	Alternatives	PRIORITY	COSTS	DV's	F.Benefit	F.Cost
2						
3	Proposal One	0.109	2,000	1	0.109	2000
4	Proposal Two	0.096	1,622	1	0.096	1622
5	Proposal Three	0.119	1,515	1	0.119	1515
6	Proposal Four	0.122	1,280	1	0.122	1280
7	Proposal Five	0.014	7,500	0	0	0
8	Proposal Six	0.013	4,375	0	0	0
9	Proposal Seven	0.115	3,187	1	0.115	3187
10	Proposal Eight	0.092	565	1	0.092	565
11	Proposal Nine	0.103	4,000	0	0	0
12	Proposal Ten	0.113	3,300	1	0.113	3300
13	Proposal Eleven	0.103	5,086	0	0	0
14				Totals:	0.766	13469

Figure 13 – Solver Results

The results are exactly the same as those when we sorted by benefit/cost ratio (which you can easily do in this spreadsheet as well). Is this true in general? Usually, but not always. It will be true if, the available funding (in this case $15,000) is exactly equal to one of the values in the cumulative cost column of the sorted benefit/cost display. If this is not the case, the organization might be better off (realize a greater total benefit) by replacing one or more higher benefit/cost ratio projects with two or more lower benefit/cost ratio projects.

We can perform the above optimization for different values of available funding and plot the results. To do this, we first add a cell to the Excel spreadsheet representing the limit on available funding and modify the constraint to reference this cell instead of the $15000 as we did earlier. See Figure 14 and Figure 15.

If we first enter a very large limit for the funds available, we get the

Alternatives	PRIORITY	COSTS	DV's	F.Benefit	F.Cost	
Proposal One	0.109	2,000	1	0.109	2000	
Proposal Two	0.096	1,622	1	0.096	1622	
Proposal Three	0.119	1,515	1	0.119	1515	
Proposal Four	0.122	1,280	1	0.122	1280	
Proposal Five	0.014	7,500	0	0	0	
Proposal Six	0.013	4,375	0	0	0	
Proposal Seven	0.115	3,187	1	0.115	3187	
Proposal Eight	0.092	565	1	0.092	565	
Proposal Nine	0.103	4,000	0	0	0	
Proposal Ten	0.113	3,300	1	0.113	3300	
Proposal Eleven	0.103	5,086	0	0	0	
			Totals:	0.766	13469	15,000
						Available

Figure 14 – Cell With Funding Limit

solution shown in Figure 16 in which all proposals are funded for a total cost of $34,430. Next, changing the amount available to be 1 dollar less than 34,430, we get the solution shown in Figure 17 in which all but Proposal five is funded for a total cost of $26,930.

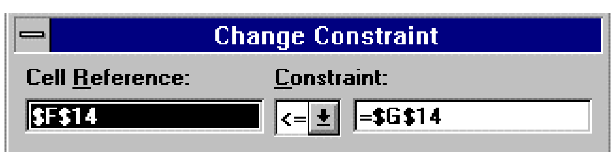


Figure 15 – Constraint for Funding Limit

Alternatives	PRIORITY	COSTS	DV's	F.Benefit	F.Cost	
Proposal One	0.109	2,000	1	0.109	2000	
Proposal Two	0.096	1,622	1	0.096	1622	
Proposal Three	0.119	1,515	1	0.119	1515	
Proposal Four	0.122	1,280	1	0.122	1280	
Proposal Five	0.014	7,500	1	0.014	7500	
Proposal Six	0.013	4,375	1	0.013	4375	
Proposal Seven	0.115	3,187	1	0.115	3187	
Proposal Eight	0.092	565	1	0.092	565	
Proposal Nine	0.103	4,000	1	0.103	4000	
Proposal Ten	0.113	3,300	1	0.113	3300	
Proposal Eleven	0.103	5,086	1	0.103	5086	
			Totals:	0.999	34430	99,999
						Available

Figure 16 – Solution for Very Large Limit

Alternatives	PRIORITY	COSTS	DV's	F.Benefit	F.Cost	
Proposal One	0.109	2,000	1	0.109	2000	
Proposal Two	0.096	1,622	1	0.096	1622	
Proposal Three	0.119	1,515	1	0.119	1515	
Proposal Four	0.122	1,280	1	0.122	1280	
Proposal Five	0.014	7,500	0	0	0	
Proposal Six	0.013	4,375	1	0.013	4375	
Proposal Seven	0.115	3,187	1	0.115	3187	
Proposal Eight	0.092	565	1	0.092	565	
Proposal Nine	0.103	4,000	1	0.103	4000	
Proposal Ten	0.113	3,300	1	0.113	3300	
Proposal Eleven	0.103	5,086	1	0.103	5086	
			Totals:	0.985	26930	34,429
						Available

Figure 17 – Solution for Limit of 34,429

If we continue to change the amount available to be one dollar less than the total cost of the previous solution, we will generate the series of points shown in Figure 18. Given a level of available funding, the best solution is that point at, or to the left of the available funding level.

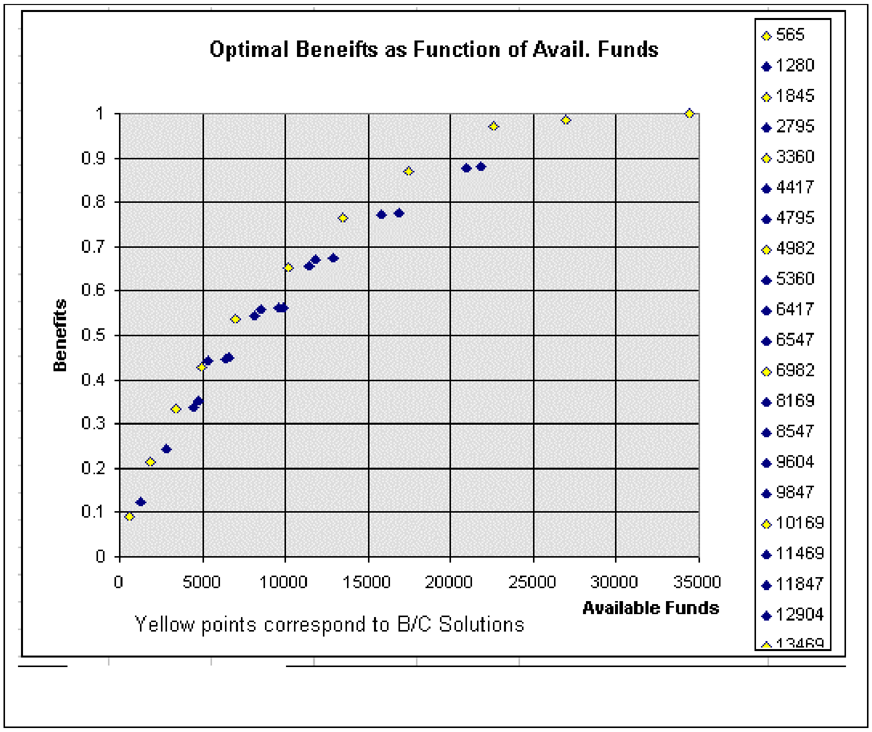


Figure 18 – Optimal Benefits vs. Available Funding

The yellow points in Figure 18 correspond to the points on the Benefit/Cost efficient frontier in Figure 7. The points in benefit optimization plot that do *not* correspond to any points in benefit/cost efficient frontier will be 'hidden' when making a resource allocation using benefit/cost. This may or may not be significant. Suppose, for example, $1300 of funding was available. The benefit maximization method yields a benefit of .122 at a cost of $1280, while the benefit/cost method yields a benefit of .092 at a cost of $565. Which solution is better, the solution produced by the method that maximizes the cumulative benefit/cost ratio or the solution produced by the method that maximizes total benefit? Is the 56% increase in costs justified by the 25% increase in benefits? If the amount expended doesn't matter[14], then any increase in total benefit is an improvement and the solution that maximizes total benefit is better. However, the benefit/cost ratio of the maximum benefit solution is smaller, and this may seem 'sub-optimal' to those who have been taught to maximize the benefit/cost ratio. The danger of maximizing the benefit/cost ratio when benefit is more appropriate is well illustrated with the following example.

Consider a situation where you can invest in either of two business opportunities: (A) you can invest one dollar and get ten dollars back, and (B) you can invest 50 dollars and get 100 dollars back. However, you have only 50 dollars to invest. Both the benefit/cost solutions and the maximum

Alternatives	PRIORITY	COSTS	dealDVs	F.Benefit	F.Cost	
Small investment	0.1	1	0	0	0	
Large investment	1	50	1	1	50	
			Totals:	1	50	50.00
	Small		1	0.2		
	Large		50	1		
	S & L		51	1.2		

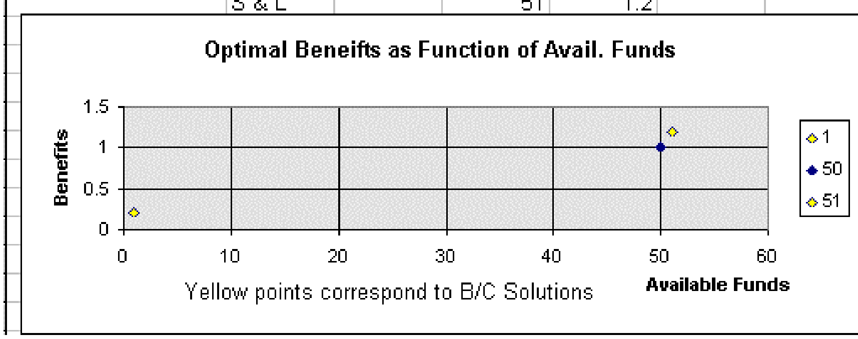


Figure 19 – Optimization vs. Benefit/Cost

[14] As an aside, there are organizations that prefer to spend as much of their budget as possible to prevent having the budget decreased the following year.

benefit solutions as a function of the amount to invest are plotted in Figure 19. While all three solutions points are achieved following a maximum of benefits, only the two points shown in yellow are achieved following a maximization of benefit/cost. In particular, if 50 dollars is available for investing, the maximization of benefit solution is to invest $50 (recommending that $50 be invested for the $100 return), while the nearest point in yellow to the $50 for the maximization of benefit/cost is at $1 (suggesting that $1 be invested for a $10 return).

A variation of this situation is that there is no limit on available funding, but you must choose either (A) or (B), but not both. If you had to make (and possibly justify) the decision based on benefit/cost ratios, you would choose the first proposition, because a benefit/cost ratio of 10 is better than a benefit/cost ratio of two. However, the maximization of benefit solution (shown in Figure 20) is to choose the second proposition wherein you

1	Alternatives	PRIORITY	COSTS	ChoiceDV:	F.Benefit	F.Cost
2						
3	Small investment	0.2	1	0	0	0
4	Large investment	1	50	1	1	50
5				Totals:	1	50
6						
7		Either small or large		1		

Figure 20 – Optimum is Large Investment

would walk away with a 50-dollar profit rather than a 9-dollar profit. Clearly, he benefit/cost ratio approach produces the wrong answer. In general, whenever unexpended funds from the resource allocation are not relevant, the benefit/cost ratio approach is not applicable.

Another example of the inappropriateness of maximizing the benefit/cost ratio is related to the knapsack problem designation given to this type of optimization. Suppose a camper is planning to take a knapsack of books on a trip. The knapsack can only hold a certain volume of books and each book has a different expected benefit15 to the camper. Which books should the camper take so as to maximize the total benefit? There will likely be some slack or unused volume in the knapsack., but slack is not one of the campers objectives. The camper 's objective is clearly to maximize the value of the books. The ratio of the total benefits of the selected books to the amount of volume used is really not of any significance. If, after tentatively selecting the books, the knapsack were replaced with one a bit larger, the camper might be able to remove a previously selected book and replace it with a larger book that has more value but a lower benefit/volume ratio. This would increase the campers overall objective, maximizing benefit. The increase in occupied volume from replacing the smaller book by the larger is of no consequence so long as the new volume constraint is not violated.

The problem can be made more complex by introducing other factors such as weight and /or cost. These factors can be considered as objectives and/or constraints. For example, suppose the camper had a budget of $X for books. This constraint can easily be added to the mathematical programming problem along with the volume constraint and an optimal solution (combination of books) can be found. This optimal will maximize the total value of the books without exceeding either the volume or cost constraints.

Would cost also be an objective in addition to a constraint? It depends on the situation. If the camper were deciding which books to take from his existing library, cost would *not* be an objective. If the camper were told he could spend up to but no more than $X to purchase books, then cost would *not* be an objective. If the camper were given $X from which he could purchase books, with any remaining funds retained by the camper, then cost *would be* an objective.

The assumptions necessary to formulate the resource allocation to fit a real world situation can best be identified by focusing on objectives. In particular, answering the question of whether cost is an objective as well as a constraint can best be answered by asking if cost, or the unexpended funds of the resource allocation, have significance to the decision makers. The

[15] The expected benefits can be derived with an AHP model.

following are three examples where cost (or unexpended funds) would not be considered as an objective:

Military campaign: In a military campaign, the objective is to get maximum military effectiveness; dollars not spent are virtually meaningless. Would military decision-makers be concerned with unexpended money if that would in any way decrease their effectiveness (even though the overall benefit/cost ratio is decreased)?

Small business competition: Suppose a small business in a competitive environment has extended its credit to limit. The managers feel they need to be as competitive as possible. Any increase in competitiveness is worth the extra cost, provided money is available. Having funds unexpended is really not part of the objective.

Government department's budget: money not spent can't be spent the next year[16].

Cost as a constraint and/or an objective:

When allocating resources, it is usually obvious when cost is a constraint – e.g., an organization is constrained to work within a given budget (although they can often try to justify increases to the budget). It is not so obvious, when allocating resources, if cost is also an objective.

What is cost anyway? A primitive society without money will engage in trade with barter. An advanced society has markets that, in equilibrium, equate cost with value. Two people trading hand made goods in a primitive society might take the following factors (or objectives) into account when contemplating a trade:

1. production time
2. value of component parts
3. tangible benefits
4. intangible benefits (e.g., artistic value)
5. value for future trades (e.g., have monetary value, such as art).

---

[16] An interesting twist to this situation: not only is any money not spent *not* carried over to next year, but the under-spending is likely to result in a *decrease* in next year's budget. Now cost again becomes an objective, but the objective is to spend as much as possible!

In a sense, the value (or cost) of something being traded is a synthesis of objectives. More advanced societies attempt to translate this value into dollars with monetary and market systems. But there may not be a market to place a value on a proposed new weapon system. Or, even if a market does exist, the market value may not represent the value to an individual or organization. Hence the need for organizations to be able to synthesize multiple objectives when allocating resources. It does not necessarily follow that the 'market value' or cost of an alternative is commensurate with the value of that alternative to an organization.

How to proceed if costs (or unexpended funds) are an objective as well as a constraint:

Cost can be included in the hierarchy so that the total benefit of each alternative includes the objective of low cost. (Judgments would be made comparing the relative importance of cost and the other factors, e.g., author's reputation, interest in the subject of the book, etc.; intensities defined and prioritized for the price objective, and ratings made for the cost of each book.) This is analogous to including cost along with other objectives such as style, comfort, and performance when choosing a car.

Don't include cost in the hierarchy (because sometimes people find it easier to make comparisons about benefits without having to consider cost17. Instead, optimize over a range of total budget constraint values, plotting the maximum benefit vs. cost for each of the solutions. Such a plot is called an "efficient frontier". The decision-maker examines this efficient frontier and decides which point is most desirable. This approach is preferred by those who feel that decision-makers sometimes find it easier to make judgments about objectives without considering costs.

Still another way to address cost as an objective is to add the percentage of unused budget as a variable in the objective function. The coefficient of this variable must either be estimated (which may be difficult to do) or treated as a parameter.

---

[17] Although when selecting a car, people don't seem to have any difficulty with low price as one of the objectives.

Flexibility of Benefits Optimization Approach:

The benefits optimization approach to resource allocation is extremely flexible. Constraints can be added to fit almost any managerial need. For example, The Air Force Cost Center added constraints to guarantee that each division received some minimum amount of funding. The Northeast Fisheries Center added constraints to regulate the rate of change in budget cutbacks so that members of the organization would not feel that the rug was about to be pulled out. They added a constraint for each division that guaranteed that the division total funding would be at least X% of the prior years funding, and solved the optimization of benefits for different values of X. The larger the value of X, the smaller the maximum will be. The smaller the value of X, the more flexibility there is to increase funding in areas that contribute more to the organization's objectives. Management then examined the plot of total benefit versus guarantee percent to determine what the percent should be and implemented the corresponding resource allocation. Resource allocation must be a dynamic, iterative process. By examining intermediate results, and modifying either or both of the AHP hierarchy or optimization formulation, an organization can express its unique musts and wants to find an allocation of resources that is rational and makes sense.

Constraints for dependencies

Two **mutually exclusive activities** i and j can be represented by a constraint such as:

$$X_i + X_j <=1$$

This can easily be extended to three or more mutually exclusive activities:

$$X_i + X_j +X_k + \ldots <=1$$

A constraint to specify that **one from a set** of activities must be selected can be modeled by including a mutually exclusive constraint as above, plus a constraint of the form:

$X_i + X_j + X_k + \ldots = 1$

Constraints representing Synergy

Saaty and Peniwati[18] have illustrated how constraints can be used to represent both positive and negative synergy. Consider the formulation presented above:

Maximize $.109X_1 + .096X_2 + .119X_3 + \ldots\ldots + .103X_{11}$ (Benefits)

subject to: $2000X_1 + 1622X_2 + 1515X_3 + \ldots\ldots + 5000X_{11} <= 15000$ (Cost)

where: $X_1, X_2, X_3, \ldots\ldots X_{11} >= 0$

and $X_1, X_2, X_3, \ldots\ldots X_{11} <= 1$
and $X_1, X_2, X_3, \ldots\ldots X_{11}$ are integer.

Suppose a group of activities (for example A_1 and A_2) taken together contribute more than the sum of the activities taken separately. Define the combined activity $\mathbf{A_{12}}$ to be $\mathbf{A_1}$ and $\mathbf{A_2}$ together. Suppose this activity were added to the Expert Choice model and the derived priority is **.250**. This would represent a positive synergy since individually activities A_1 and A_2 contribute .109 + .096 = .205. A negative synergy, perhaps due to overlap of contribution, would be indicated by a priority for A_{12} of less than .205. The optimization can be formulated by adding a decision variable, call it $Z_{1,2}$, where a resulting value of $Z_{1,2} = 1$ means that the combination of A_1 and A_2 is to be included. However we must add additional constraints to assure what Saaty refers to exclusion and duplication.

Exclusion means that if an activity is performed exclusively (alone), no other combination containing the activity can be performed. Thus, for each activity that is a member of a combination, the decision variable corresponding to the activity performed alone plus all other decision variables representing the combinations does not exceed 1.

Duplication assures that for each combination, there is no other combination of variables that duplicates the combination.

---

[18] Saaty, T.L. and Peniwati, K. "The Analytic Hierarchy Process and Linear Programming in Human Resource Allocation", *Proceedings of the Fourth International AHP Symposium***,** Vancouver, 1996.

Thus, for the previous example, we would have:

Maximize $.109X_1 + .096X_2 + .119X_3 + ... + .103X_{11} + \mathbf{.250\ Z_{1,2}}$ (Benefits)

subject to:

$2000X_1 + 1622X_2 + 1515X_3 + ... + 5000X_{11} \mathbf{+ 3622\ Z_{1,2}} <= 15000$ (Cost)

and constraints for exclusivity:

$X_1 + Z_{1,2} <=1$ for activity 1 with combination $Z_{1,2}$
$X_2 + Z_{1,2} <= 1$ for activity 2 with combination $Z_{1,2}$

and a constraint to prevent duplication:

$X_1 + X_2 <= 1$
to prevent duplication of combination $Z_{1,2}$.
where: the decision variables are non negative integers:

$X_1, X_2, X_3, ... \mathbf{Z_{1,2}} >=0$

$X_1, X_2, X_3, ... \mathbf{Z_{1,2}} <=1$
$X_1, X_2, X_3, ... \mathbf{Z_{1,2}}$ are integer.

Another, perhaps simpler representation of constraints representing Synergy

Identify each set of activities with synergistic effect (either positive or negative). For each set, define up to 2^n -1 decision variables representing all meaningful combinations of the activities. The cost and benefit of each possible combination represent the synergistic cost and benefit respectively. Only one constraint must be added for each set of synergistic activities – the sum of the 2^n variables must = 1. This approach results in more decision variables but fewer constraints. To apply this technique to the example above, lets represent the combinations of activity 1 and activity 2 by the four decision variables $Z_{0,0}$, $Z_{0,1}$, $Z_{1,0}$, and $Z_{1,1}$, where the first subscript refers to activity 1, the second subscript refers to activity 2, and a 0 for the subscript means that the activity is not included while a 1 means that it is included. So, for example, if $Z_{1,1}$ is 1, then both activity 1 and activity 2 are to be undertaken. The formulation using these variables would be:

Maximize $.109Z_{1,0} + .096Z_{0,1} + \mathbf{.250\ Z_{1,1}} + .119X_3 + ... + .103X_{11}$ (Benefits)
subject to:
$2000\ Z_{1,0} + 1622\ Z_{0,1} + \mathbf{3622\ Z_{1,1}} + 1515X_3 + ... + 5000X_{11} <= 15000$ (Cost)
and the additional constraint:

$\mathbf{Z_{0,1} + Z_{1,0} + Z_{1,1} <=1}$
where the decision variables are non-negative integers:
$Z_{0,1}, Z_{1,0}, Z_{1,1}, X_{3,} .... X_{11} >=0$
$Z_{0,1}, Z_{1,0}, Z_{1,1}, X_{3,} .... X_{11} <=1$
$Z_{0,1}, Z_{1,0}, Z_{1,1}, X_{3,} .... X_{11}$ are integer.

A more elaborate example

Suppose, as a more elaborate example, an additional synergistic combination of $\mathbf{A_2}$ and $\mathbf{A_3}$ produces a combination represented by decision variable $\mathbf{Z_{2,3}}$, with a benefit of **.275**. Furthermore, an additional synergistic combination of $\mathbf{A_1}$**,** $\mathbf{A_2}$ and $\mathbf{A_3}$ produces a combination represented by decision variable $\mathbf{Z_{1,2,3}}$ with a benefit of **.400**.

Maximize $.109X_1 + .096X_2 + .119X_3 + ... + .103X_{11} + \mathbf{.250\ X_{12} + .275\ Z_{1,3} + .400\ Z_{1,2,3}}$ (Benefits)

subject to:

$2000X_1 + 1622X_2 + 1515X_3 + ... + 5000X_{11} + \mathbf{3622\ X_{12} + 3137\ Z_{1,3} + 5137\ Z_{1,2,3}} <= 15000$ (Cost)

and constraints for exclusivity:

$X_1 + Z_{1,3} <=1$ for activity 1 with combination X_{12}
$X_1 + Z_{1,2,3} <=1$ for activity 1 with combination X_{14}
$X_2 + Z_{1,3} <=1$ for activity 2 with combination X_{13}
$X_2 + Z_{1,2,3} <= 1$ for activity 2 with combination X_{14}

and a constraints to prevent duplication**:**

$X_1 + X_2 <= 1$ to prevent duplication of combination X_{12}.
$X_2 + X_3 <= 1$ to prevent duplication of combination X_{13}.
$X_1 + X_2 + X_3 <= 1$ to prevent duplication of combination X_{14}.
$X_{12} + X_3 <= 1$ to prevent duplication of combination X_{14}.
$Z_{1,3} + X_1 <= 1$ to prevent duplication of combination X_{14}.

where the decision variables are non-negative integers:
$X_1, X_2, X_3, ... , \mathbf{Z_{1,3}, Z_{1,2,3}} >=0$
$X_1, X_2, X_3, ..., \mathbf{Z_{1,3}}, \mathbf{Z_{1,2,3}} <=1$
$X_1, X_2, X_3, ... \mathbf{Z_{1,3}}, \mathbf{Z_{1,2,3}}$ are integer.

The alternative formulation for this more elaborate example would be:
Define $2^3 -1$ or seven decision variables representing the combinations of activities 1, 2 and 3, where $Z_{1,0,0}$ represents doing activity 1 but not two or three, $Z_{0,1,0}$ represents doing activity 2 but not 1 or 3, $Z_{1,1,0}$ represents doing activities 1 and 2, but not 3, and so on. The formulation would be:

Maximize $.109Z_{1,0,0} + .096Z_{0,1,0} + .119Z_{0,0,1} + \mathbf{.250\ Z_{1,1,0} + .275\ Z_{1,0,1} + .400\ Z_{1,1,1}}...... + .103X_{11}$
subject to:

2000 $Z_{1,0,0}$ + 1622 $Z_{0,1,0}$ + 1515 $Z_{0,0,1}$ +**3622 $Z_{1,1,0}$ + 3137 $Z_{1,0,1}$ + 5137 $Z_{1,1,1}$** ..... + 5000X_{11} <= 15000 (Cost)

$$Z_{1,0,0} + Z_{0,1,0} + Z_{0,0,1} + Z_{1,1,0} + Z_{1,0,1} + Z_{0,1,1} + Z_{1,1,1} <= 1.$$

where the decision variables are non-negative integers:
$Z_{1,0,0}$, $Z_{0,1,0}$,…$Z_{1,1,1}$, X_4,…X_{11} >=0
$Z_{1,0,0}$, $Z_{0,1,0}$,…$Z_{1,1,1}$, X_4,…X_{11}<=1
$Z_{1,0,0}$, $Z_{0,1,0}$,…$Z_{1,1,1}$, X_4,…X_{11} are integer.

An example with two sets of synergistic activities.

Suppose activities A_1, …. A_{100} were being considered. Further suppose there are two sets of synergistic activities, the first set including activities A_1, A_2, A_3, and A_4 and a second set including activities A_8, A_9, and $A_{10.}$

For the first set, suppose the following combinations of A_1 through A_4 are possible: each activity by itself, or the following synergistic combinations (either positive or negative synergy):

A_1 with A_2

A_2 with A_3 with A_4

For this first set, define a subset of 2^4 - 1 or 15 variables:

$Z_{1,0,0,0}$, which, if set to 1, signifies A_1, but not A_2, A_3, or A_4

$Z_{0,1,0,0}$, which, if set to 1, signifies A_2, but not A_1, A_3, or A_4

$Z_{0,0,1,0}$, which, if set to 1, signifies A_3, but not A_1, A_2, or A_4

$Z_{0,0,0,1}$, which, if set to 1, signifies A_4, but not A_1, A_2, or A_3

$Z_{1,1,0,0}$, which, if set to 1, signifies A_1 and A_2 but not A_3, or A_4

$Z_{0,1,1,1}$, which, if set to 1, signifies A_2, A_3, A_4, but not A_1

- These variables are included in the objective function and functional constraints with the appropriate benefit and cost coefficients. Additionally, a constraint:

$Z_{1,0,0,0} + Z_{0,1,0,0} + Z_{0,0,1,0} + Z_{0,0,0,1} + Z_{1,1,0,0} + Z_{0,1,1,1} <= 1$ insures that only one of the combinations will be selected.

For the second set, suppose the following combinations of A_8 through A_{10} are possible: each activity by itself, or the following synergistic combinations (either positive or negative synergy):

A_8 with A_9 with A_{10}

A_9 with A_{10}.

For this first set, define a subset of 2^3 - 1 or 7 variables:

$W_{1,0,0}$, which, if set to 1, signifies A_8, but not A_9, or A_{10}

$W_{0,1,0}$, which, if set to 1, signifies A_9, but not A_8, or A_{10}

$W_{0,0,1}$, which, if set to 1, signifies A_{10}, but not A_8, or A_9

$W_{1,1,1}$, which, if set to 1, signifies A_8, with A_9 and A_{10}

$W_{0,1,1}$, which, if set to 1, signifies A_9 with A_{10} but not A_8.

These variables are also included in the objective function and functional constraints with the appropriate benefit and cost coefficients. Additionally, a constraint:

$W_{1,0,0} + W_{0,1,0} + W_{0,0,1} + W_{1,1,1} + W_{0,1,1} <=1$ insures that only one of the combinations will be selected.

Constraints for different types of 'costs'

Additional constraints can easily be added for other 'costs'. For example there may be one constraint for dollar costs, another for personnel, and still another for space. Each such constraint can also be specified for multiple time periods.

Orders of Magnitude Considerations

Care must be taken when allocating resources to alternatives that differ in cost by several orders of magnitude. If for example, the cost for some alternatives are in the 1-10 thousand dollar range while the cost for others are in the hundreds of millions of dollars, then the benefit measure must be capable of spanning a commensurate range ratio. This would not be the case with a narrow hierarchy (one that doesn't encompass a wide range of objectives) or intensities that do not span ranges that can adequately reflect the difference in contribution of very inexpensive alternatives and very expensive alternatives. One approach, then, is to construct an evaluation hierarchy that is both broad -- so that expensive alternatives can be recognized for their contributions over a broad range of sub-objectives while inexpensive alternatives will be recognized only for the more narrow areas to which they contribute. The evaluation hierarchy should also contain numerous rating intensities so as to span as wide a range of measure as possible, perhaps even two orders of magnitude.

It may be difficult or impractical to do this in some circumstances, so another approach, and one that is similar to the traditional way organizations allocate funds hierarchically, is to divide the alternatives into categories from low cost to high cost and allocate separately within each category. For example, if projects are being considered that range in cost from 5 thousand to 8 million dollars, the projects can be organized into categories 1-10K, 10K-100K, 100K-1M and 1M-8M. The question then arises as to how much of the total budget is to be allocated to each category. This can be addressed various ways. One would be with a hierarchy to derive priorities for each category's contribution to the organizations objectives. The total funding can then be multiplied by the resultant priorities to determine how much will be allocated within each category. Another way would be to allocate funding to each category based on the ratio of the total cost of the projects in each category to the total costs of all projects, and multiply the funding amount in each category by the ratio of the total funding available to the total cost of all projects.

Summary of B/C Ratios vs. Optimization of Benefits:

B/C Ratios:

Cost is an objective because it goes into B/C ratio.

If cost (or unexpended funds) is not an objective, then B/C ratio should not be used.

examples: Military, Competitive business; government agency

Pros: Easy to Solve.

Cons: Can lead to wrong answers if applied blindly (assumptions difficult to verify).

Can only deal with one constraint.

Optimization of Benefits:

Cost may not be an objective, or
Cost is an objective. If so, can do one of the following:
1. include cost as an objective in the hierarchy, or
2. consider cost as a constraint and optimize benefits over a range of total cost constraint values, plotting the maximum benefit vs. cost for each of the solutions. Such a plot is called an "efficient frontier". The decision maker examines this efficient frontier and decides which point is most desirable; or
1. add percentage of unused budget as variable in optimization the objective function.

Pros: Easy to verify assumptions
Can include many types of constraints
Very flexible
Cons: Computationally difficult
May have to explain why replacing a higher b/c alternative with a more costly but lower b/c alternative as budget is increased, is in the best interests of the organization.

Comparison of results from maximizing b/c ratio and maximizing benefits:

- Results are often quite close
- Identical results if budget limit corresponds to a cumulative cost break value
- Drastically different results are possible, especially in situations where cost, or unexpected funds are not important.

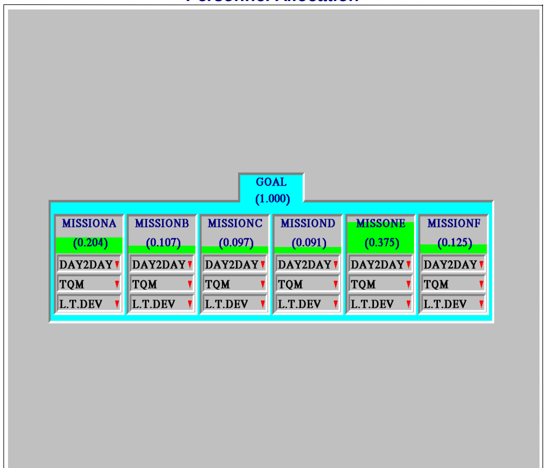
Activity Level Resource Allocation

Whereas in discrete alternative resource allocation, each alternative (e.g., research proposal) is discrete from (although possibly related to) the other alternatives, activity level resource allocation considers one or more *levels* of funding for each alternative (e.g., department). For example, each department in an organization can be considered for funding at current funding levels, +/- 5%, +/- 10%, etc. A plan is developed specifying the type and extent of activities each department would perform at each level of funding. An AHP model is used to evaluate the expected contribution to the organization's objectives for each alternative level of funding. A prototype of an activity level resource allocation problem will be used to illustrate a heuristic incremental benefit/cost approach as well as a maximization of total benefit approach.

Consider an organization with four departments, Marketing, Production, Operations, and Maintenance. Each department submits plans for how they would operate with a bare minimum number of people, with the current number of people, and with a moderate increase from current funding level. The organization has six major missions, which we will refer to as Mission A through Mission F. To keep the example simple, we will assume that the objectives within each mission include responding to day to day needs and activities, total quality efforts, and long term development. An Expert Choice model with mission priorities derived from pairwise comparisons is shown in Figure 21.

Intensities were defined for rating how much each alternative level of funding for each department would contribute to each of the objectives within each mission. A portion of the RATINGS model is shown in Figure 22. Notice that each 'line item' is a level of funding within a department. Departments are identified by a common alphanumeric prefix before the delimiter, in this case a colon. The costs are shown in thousands of dollars. The Total column represents the un-normalized total expected contribution of that line item (department at a specified level of funding) toward the organization's overall objectives. Objectives for which no contribution is expected at a specified level of funding are left blank.

Personnel Allocation



Abbreviation	Definition
DAY2DAY	Responding to day to day needs and activities
L.T.DEV	Long term development
MISSIONA	
MISSIONB	
MISSIONC	
MISSIOND	
MISSIONF	
MISSONE	
TQM	Total quality efforts

(Priorities shown are 'Local' -- relative to parent node.)

Figure 21 – Organizational Objectives

MISSIONA- DAY2DAY

EXTREME	SIGNIFIC	LARGE	MODERATE	TAD
1 (1.000)	2 (.483)	3 (.276)	4 (.152)	5 (.028)

	Alternatives	TOTAL	COSTS	MISSIONA- DAY2DAY .0680	TQM .0680	L.T.DEV .0680	MISSIONB- DAY2DAY .0358	TQM .0358	L.T.DEV .0358	MISSIONC- DAY2DAY .0324	TQM .0324
1	Marketing: 1 Person	0.055	80	MODERATE			MODERATE			LARGE	
2	Marketing: 2 People	0.342	160	LARGE	LARGE	MODERATE	LARGE	LARGE	LARGE	LARGE	SIGNIFIC
3	Production: 2 People	0.062	140	MODERATE			MODERATE			TAD	
4	Production: 3 People	0.260	210	LARGE	LARGE	MODERATE	MODERATE	LARGE	SIGNIFIC	MODERATE	LARGE
5	Production: 4 People	0.406	280	LARGE	LARGE	LARGE	LARGE	SIGNIFIC	MODERATE	LARGE	LARGE
6	Operations: 3 People	0.058	225	SIGNIFIC			LARGE			MODERATE	
7	Operations: 4 People	0.290	300	EXTREME	MODERATE	LARGE	SIGNIFIC	SIGNIFIC	MODERATE	LARGE	LARGE
8	Operations: 5 People	0.408	350	EXTREME	LARGE	SIGNIFIC	EXTREME	SIGNIFIC	LARGE	LARGE	LARGE
9	Operations: 6 People	0.547	400	EXTREME	LARGE	SIGNIFIC	EXTREME	SIGNIFIC	SIGNIFIC	SIGNIFIC	SIGNIFIC
10	Maintenance: 3 People	0.063	90	LARGE			LARGE			MODERATE	
11	Maintenance:4 People	0.305	120	SIGNIFIC	MODERATE	SIGNIFIC	LARGE	LARGE	MODERATE	LARGE	LARGE
12	Maintenance: 5 People	0.416	150	EXTREME	LARGE	LARGE	LARGE	SIGNIFIC	SIGNIFIC	LARGE	LARGE
13	Maintenance: 6 People	0.766	180	EXTREME	EXTREME	EXTREME	SIGNIFIC	SIGNIFIC	SIGNIFIC	SIGNIFIC	SIGNIFIC
14											

Figure 22 – Activity Level Ratings

Benefit/Cost Ratios

Because only one level can be selected for each department, the maximization of cumulative benefit/cost ratios is not nearly as straightforward as for the discrete resource allocation situation. The following heuristic approach has been found to provide a reasonably good allocation. We begin by funding each department at its lowest level of funding, thereby producing a combination with the lowest total benefit and lowest total cost. We look for the best choice of department to increase to its next level of funding. We make the choice by choosing the department with the highest incremental benefit/cost ratio from its present level of funding. We continue doing this until we exceed the total budget, and then go back to the last level of increase.

This method will not necessarily result in the maximum benefit subject to the total budget constraint, but it is likely to be quite close. One of the practical difficulties in implementing the method occurs when a department's contribution fails to follow a diminishing marginal rate of return curve. Theoretically, the law of diminishing returns states that the incremental benefit from an increase in one unit of cost should be a non-increasing function. If this assumption is violated, then a relatively low incremental benefit/cost ratio at some level of funding can 'hide' a higher incremental benefit/cost ratio from being seen by the algorithm. To avoid this difficulty, a recursive procedure is used to remove the lower incremental benefit/cost level(s) from consideration.

The initial lowest cost allocation consists of the lowest levels for each department as shown in Figure 23, Figure 24, and Figure 25.

The values in the Benefit column in Figure 23 are normalized equivalents of the total values in Figure 22 – normalized so that, if each department were funded at the maximum level, the total would be 100. The >>> symbol in Figure 25 suggests that if additional funds beyond the $535K are to be allocated, then the Maintenance department should be increased to a higher level of funding.

	Activity	B	C
1	Marketing: 1 Person	2.648	80.000
2	Marketing: 2 People	16.595	160.000
3	Production: 2 People	3.014	140.000
4	Production: 3 People	12.602	210.000
5	Production: 4 People	19.712	280.000
6	Operations: 3 People	2.832	225.000
7	Operations: 4 People	14.050	300.000
8	Operations: 5 People	19.781	350.000
9	Operations: 6 People	26.551	400.000
10	Maintenance: 3 People	3.079	90.000
11	Maintenance:4 People	14.773	120.000
12	Maintenance: 5 People	20.172	150.000
13	Maintenance: 6 People	37.142	180.000

Figure 23 – Each Activity At Lowest Level

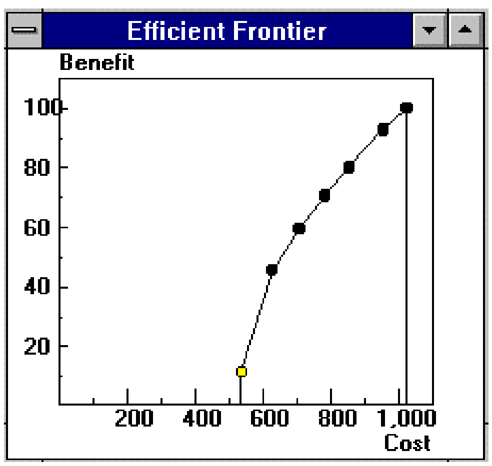


Figure 24 – Efficient Frontier

Selected

Marketing: 1 Person
Production: 2 People
Operations: 3 People
Maintenance: 3 People >>>

Total Benefit: 11.57
Total Cost : 535.

Figure 25–More Maintenance If More Funding

Suppose we didn't know this, but asked the Department managers to meet and decide. Suppose they decided to increase the Operations Department to four people. The results would be as shown in Figure 26 and Figure 27. The total benefit has increased from 11.57 to 22.79[19] and the total cost has increased from $535K to $610K.

	Activity	B	C
1	Marketing: 1 Person	2.648	80.000
2	Marketing: 2 People	16.595	160.000
3	Production: 2 People	3.014	140.000
4	Production: 3 People	12.602	210.000
5	Production: 4 People	19.712	280.000
6	Operations: 3 People	2.832	225.000
7	Operations: 4 People	14.050	300.000
8	Operations: 5 People	19.781	350.000
9	Operations: 6 People	26.551	400.000
10	Maintenance: 3 People	3.079	90.000
11	Maintenance:4 People	14.773	120.000

Figure 26 – Seat of the Pants Decision

[19] Since the benefit scale is a ratio level of measure, we can say that this is about a 100% increase in total benefit.

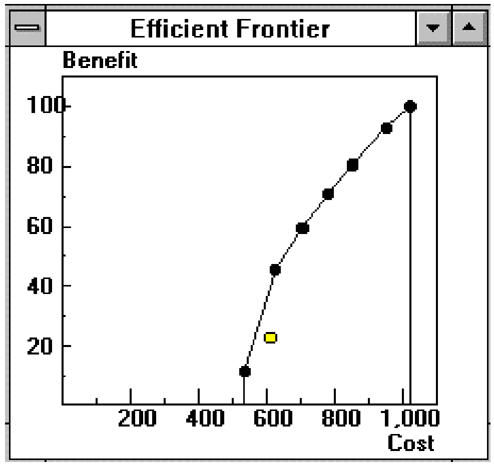


Figure 27 – Below the Efficient Frontier

	Activity	B	C
1	Marketing: 1 Person	2.648	80.000
2	Marketing: 2 People	16.595	160.000
3	Production: 2 People	3.014	140.000
4	Production: 3 People	12.602	210.000
5	Production: 4 People	19.712	280.000
6	Operations: 3 People	2.832	225.000
7	Operations: 4 People	14.050	300.000
8	Operations: 5 People	19.781	350.000
9	Operations: 6 People	26.551	400.000
10	Maintenance: 3 People	3.079	90.000
11	Maintenance:4 People	14.773	120.000
12	Maintenance: 5 People	20.172	150.000
13	Maintenance: 6 People	37.142	180.000

Figure 28 – Incremental Benefit Cost Allocation

But if we followed the algorithm, we would have increased funding in the Maintenance Department from 3 people to 6 people[20]. The results are shown in Figure 28, Figure 29, and Figure 30.

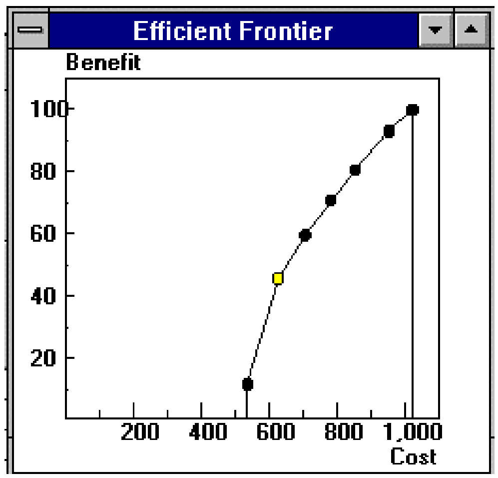


Figure 29 – Back on the Efficient Frontier

Thus, increasing funding in the Maintenance Department from 3 people to 6 people, rather than increasing the funding in the Marketing Department from 1 person to 2 people, would result in almost double the benefit (45.64 vs. 22.79) for only a slightly extra cost ($625K vs. $610K). The >>> symbols in Figure 30 indicate that the next increase should be in the Marketing Department. Following the algorithm would result in the points shown on the efficient frontier plot in Figure 29.

[20] Staffing levels of 4 and 5 were skipped because the incremental benefit/cost for this department did not follow the law of diminishing returns.

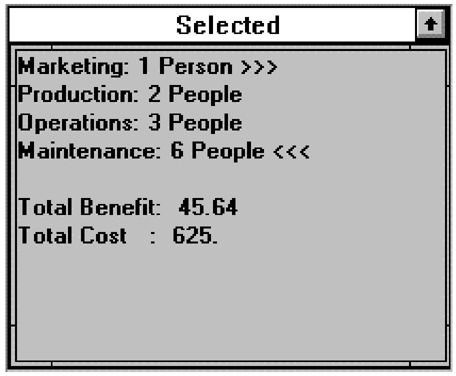


Figure 30 – Change Indicators

Maximizing Benefits -- Optimization

The problem formulation for the maximization of benefit approach that we used for the discrete alternative resource allocation extends easily to the activity level resource allocation case. We want to find that combination of activity levels that maximizes the total benefits without exceeding the given budget.[21] This problem, is formulated as a zero-one integer mathematical programming problem, sometimes referred to as a multiple choice knapsack problem. The formulation for the prototype activity level resource allocation presented above (refer to Figure 22 and Figure 23) is as follows:

Maximize $2.648X_{1,1} + 16.595X_{1,2} + 3.014X_{2,1} + 12.602X_{2,2} + 19.712X_{2,3} +$

$+ \ldots + 3.079X_{4,1} + 14.773X_{4,2} + 20.172X_{4,3} + 37.142X_{4,4}$

(Benefits)[22]

subject to: $80X_{1,1} + 160X_{1,2} + 140X_{2,1} + 210X_{2,2} + 280X_{2,3} +$

[21] Here, as in the discrete alternative resource allocation case, maximizing the cumulative benefit/cost ratio really is more straightforward and understandable than the incremental benefit/cost approach. Furthermore, it doesn't depend on questionable underlying assumptions. However, the computation complexity, even with high speed computers, is sometimes a concern.

[22] The coefficients of the objective function correspond to the normalized benefit column of Figure 23 (normalized so that the total benefit is 100 if each department were funded at the maximum level). Alternatively, the un-normalized values in the total column of Figure 22 could have been used.

	A	B	C	D	E	F	G
1	Alt/Levels	PRIORITY	COSTS	LDV's	F. Benefit	F.Cost	
2							
3	Marketing: 1 Person	2.648	80	0	0.00	0.00	
4	Marketing: 2 People	16.595	160	1	16.60	160.00	
5	Production: 2 People	3.014	140	1	3.01	140.00	
6	Production: 3 People	12.602	210	0	0.00	0.00	
7	Production: 4 People	19.712	280	0	0.00	0.00	
8	Operations: 3 People	2.832	225	0	0.00	0.00	
9	Operations: 4 People	14.05	300	1	14.05	300.00	
10	Operations: 5 People	19.781	350	0	0.00	0.00	
11	Operations: 6 People	26.551	400	0	0.00	0.00	
12	Maintenance: 3 People	3.079	90	0	0.00	0.00	
13	Maintenance:4 People	14.773	120	0	0.00	0.00	
14	Maintenance: 5 People	20.172	150	0	0.00	0.00	
15	Maintenance: 6 People	37.142	180	1	37.14	180.00	
16				Totals:	70.80	780.00	800.00
17							Available
18	Constraints to assure one and only one level for each Alternative:						
19	Marketing	1					
20	Production	1					
21	Operations	1					
22	Maintenance	1					

Figure 31–Activity Level Optimization

$+\ldots\ldots + 90X_{4,1} + 120X_{4,2} + 150X_{4,3} + 180X_{4,4} \quad\quad <= \text{Budget} \quad\quad \text{(Costs)}$

where: $X_{1,1}, X_{1,2}, X_{2,1}, \ldots\ldots X_{4,4} \quad >=0$

$X_{1,1} + X_{1,2} \quad = 1$

$X_{2,1} + X_{2,2} + X_{2,3} \quad = 1$

$X_{3,1} + X_{3,2} + X_{3,3} + X_{3.4} \quad = 1$

$X_{4,1} + X_{4,2} + X_{4,3} + X_{4.4} \quad = 1$

and $X_{1,1}, X_{1,2}, X_{2,1}, \ldots\ldots X_{4,4}$ <=1
and $X_{1,1}, X_{1,2}, X_{2,1}, \ldots\ldots X_{4,4}$ are integer.

contains an Excel spreadsheet in which the optimization is performed. The values correspond to the optimal solution for an available budget of $800K. Columns A, B, and C were copied and pasted from the Expert Choice Ratings sheet. The level decision variables (designated as LDV's) are contained in Column D. Column E contains formulas for Funded Benefits. For example, the formula in cell E3 is =D3*B3. Column F contains formulas for Funded Costs. For example, the formula in cell F3 is =D3*C3. Cell G16 contains the available funding, presently set at $800K. Cells B19 through B22 each contain the sum of the decision variables for the respective activity. For example, cell B19 contains the formula: =SUM(D3:D4).

The solver parameters are shown in Figure 32. The target cell, E16, is the sum of the funded benefits. The By Changing Cells are the level decision variables. The five constraints are as follows:

1.B19:B22 = 1 insures that one and only one level will be funded for each alternative.

2.F16<=G16 constrains total spending.

3.The level decision variables are less than or equal to 1.

4.The level decision variables are integer.

5.The level decision variables are non-negative.

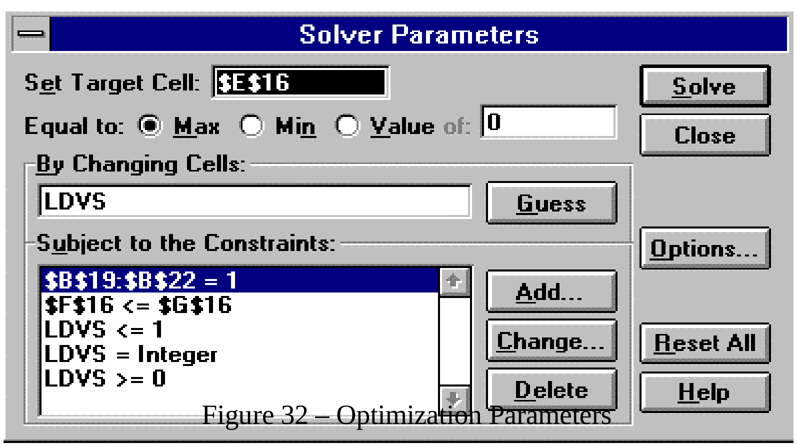


Figure 32 – Optimization Parameters

Figure 33 and Figure 34 show the resulting maximum benefit as a function of cost. The levels indicate the level of funding for each of the four activities (departments). For example, 2,1,1,4 in Figure 33 means that the first department is funded at level 2, the second and third departments at levels 1, and the fourth department at level 4. The yellow dots in Figure 34 correspond to the incremental benefit/cost solutions discussed above. Given a level of available funding, the best solution is that point at, or to the left of that funding level. Like the discrete alternative resource allocation funding problems, the difference between the maximum benefit and maximum benefit/cost solutions for activity level resource allocation problems may or may not be significant. If the solution point at or to the left of a given level of available funding is a solution point to both methods (a yellow dot) then there is no difference. Otherwise the difference is the difference between the benefit at the first maximum benefit solution point and the first maximum benefit/cost solution point to the left of the available funding. For example, if $650K of funding is available, there is no difference. However, if $600K is available, maximizing benefits produces almost three times the benefit than maximizing benefit/cost ratio (28.67 vs. 11.57).

Benefits	Costs	Levels	Corresponding IBC	
			Solution	
11.57	535	1,1,1,1	1	
23.27	565	1,1,1,2		
28.67	595	1,1,1,3		
45.64	625	1,1,1,4	2	
56.85	700	1,1,2,4		
59.58	705	2,1,1,4	3	
62.58	750	1,1,3,4		
66.44	770	1,2,2,4		
69.17	775	2,2,1,4		
70.80	780	2,1,2,4	4	
72.17	820	1,2,3,4		
76.53	830	2,1,3,4		
80.39	850	2,2,2,4	5	
83.30	880	2,1,4,4		
86.12	900	2,2,3,4		
87.50	920	2,4,2,4		
92.89	950	2,2,4,4	6	
93.23	970	2,3,3,4		
100.00	1020	2,3,4,4	7	

Figure 33 – Benefits, Costs, Activity Levels

Resource Allocation Summary

The need to synthesize information and judgments is a difficult, if not insurmountable problem when allocating resources in medium and large size organizations. High level executives can best judge the relative importance of the organization’s objectives while operational and technical personnel can best judge the performance of the alternatives with respect to these objectives. Knowledge and judgments about *both* objectives and alternatives are required in order to rationally allocate resources. One might say that this is just a communications problem, but it is more than that. No matter how many meetings and discussions are held, it is virtually impossible to combine the requisite information and judgment without the ability to properly measure and synthesize. The methodologies presented in this chapter make this possible.

The AHP process is used to derive priorities for the organization's objectives as well as the alternatives that compete for resources. Once priorities are derived, either a benefit/cost or maximization of benefit approach can be used. The former offers some computational benefits whereas the latter is more robust and flexible. We have presented methodologies for discrete alternative resource allocation as well as for activity level resource allocation. The former is particularly useful when considering new activities, such as proposals for research and development. The latter can easily be applied in any organization that does periodic resource allocation. It is particularly useful as an alternative to 'across the board cuts' when an organization is faced with budget cutbacks[23] since 'across the board cuts' will eventually lead to the demise of an organization that faces competition in a changing environment. Instead, this rational

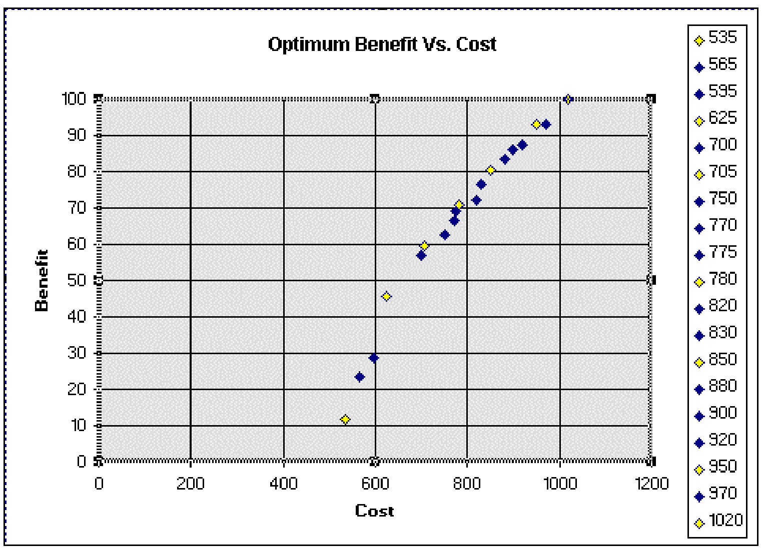


Figure 34 – Optimum Benefit Vs. Cost

[23] Although resource allocations during stable or increasing budgets do not concern management as much as cutbacks, the need to gain and maintain a competitive advantage should make this activity just as important as when budgets are being reduced.

resource allocation methodology will help identify those departments that should receive increased funding even when the overall budget is being decreased. The methodologies are extremely flexible and can be implemented piecemeal -- yielding incremental improvements over the organization's current resource allocation "methodology". Improvement may come just from prioritizing the organization's objectives, and / or from evaluating the expected contributions at different levels of funding for each department.

Implementation of the complete methodology, including the combinatorial optimization problem will require iteration so that the intermediate 'solutions' can be questioned, challenged, and modified by:

- changing the AHP model and / or
- changing judgments and / or
- adding or modifying constraints in the optimization.

The 'final' solution should not only be mathematically 'optimal', but intuitively appealing to those who will have to live and compete with the resulting levels of funding.