Chapter 10

Feedback

Intuitive and Formal Feedback

In some (but not necessarily all) choice decisions, the importance of the objectives may depend on the alternatives being considered. This dependence can be accommodated either with formal feedback calculations or, in most cases, intuitively by the decision maker(s). Consider the following example. Suppose you are the mayor of a medium size city. The city council has just approved funding for a bridge that will connect the eastern and southern districts– saving the residents 30 minutes in commuting time. You announce that the winning proposal will be chosen using a formal evaluation methodology in which the proposals will be evaluated on the basis of strength and aesthetics. In order to be fair, you will, before receiving any bids, specify which of the two objectives will be more important. It seems obvious[1] that strength is much more important than aesthetics and you publicly announce that strength will be the most important objective in choosing the winning proposal.

Subsequently, two alternative designs are proposed for the new bridge. Bridge A is extremely save (as safe as any bridge yet built in the State) and beautiful. Bridge B is twice as strong as bridge A, but is UGLY!. Your hands are tied – you have announced that the most important objective is strength and, as the example below will illustrate, you must choose the ugly bridge. The bridge is built and many town residents are reminded of your decision at least twice a day. You lose the next election and will be wary of formal evaluation methodologies for the rest of your life. Yet formal evaluation methodologies are, as we have already seen, necessary to cope with the complexity of most crucial decisions. The answer is not to avoid formal evaluation methodologies, but to use them in ways that make sense!

Evaluation methodologies that neglect the dependence of objective priorities on the alternatives being considered are sometimes mandated by

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[1]It would be difficult to defend a position that the strength of the bridge is not more important than aesthetics.

regulations. For example, government organizations sometimes have regulations that mandate that evaluation weights be established and announced before a request for proposal is issued. In our simple two objective example above, the Mayor would have had to declare that strength was more important than aesthetics and then have had to chose the ugly bridge, which, he intuitively knew was the wrong choice. But intuition alone is not adequate in real world decisions because the numerous competing factors of the decision challenge man's cognitive abilities to adequately evaluate and process the information. Hence we must rely on decision models when evaluating alternatives, but we must use them in ways that make sense – intuitively and logically. If we need to incorporate feedback between alternatives and objectives, but fail to do so, our intuition will tell us that there is something wrong with the tentative conclusion. Recognizing this, we can incorporate the necessary feedback through iteration, or through mathematical means with a 'supermatrix.' Lets see how this can be done using the bridge selection problem discussed above.

Top Down Evaluation of Bridge Selection AHP Model

A top down evaluation of an AHP model for the Mayor's simple two objective decision would proceed as follows. Before examining the alternatives, most rational people would judge safety to be much more important than aesthetics. Suppose the Mayor judged safety to be extremely more important than aesthetics as shown in Figure 1 and Figure 2.

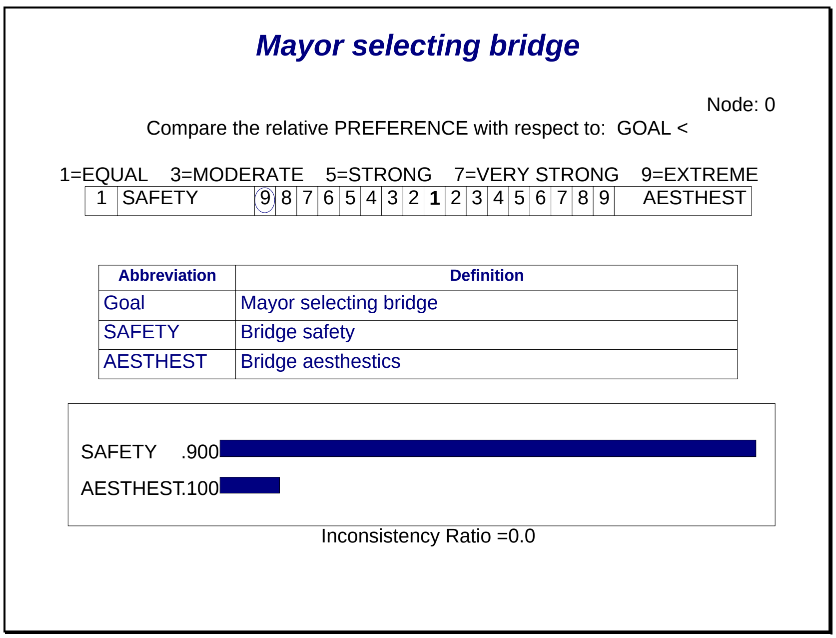

Figure 1 – Importance of Safety vs. Aesthetics

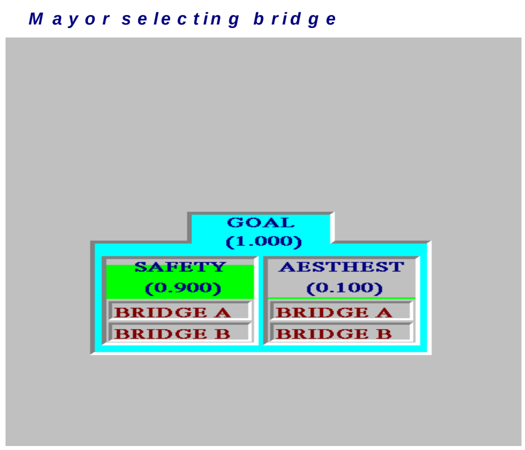

Figure 2 – Model for Bridge Selection

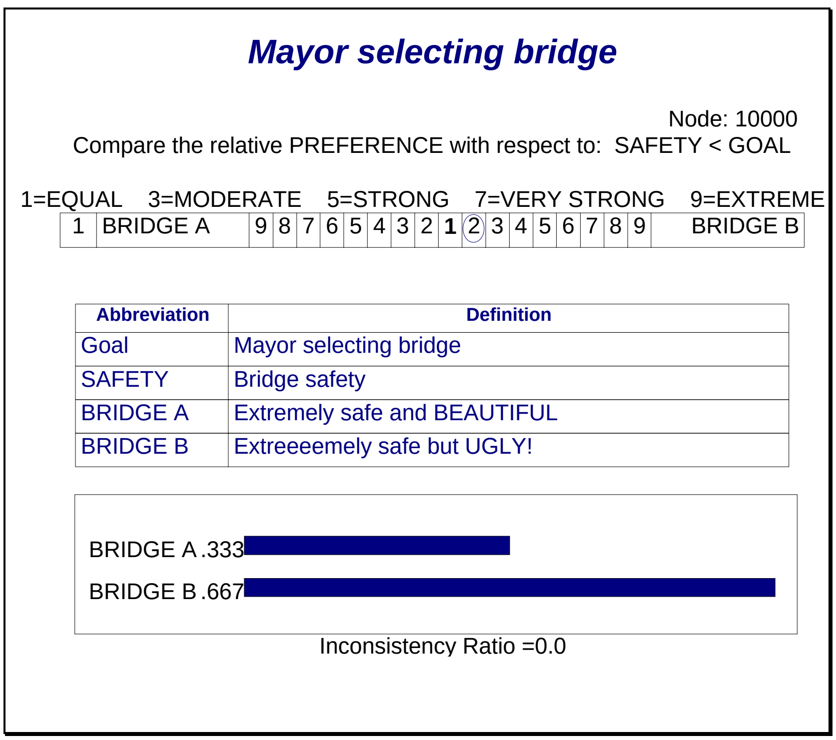

Figure 3 – Preference with respect to Safety

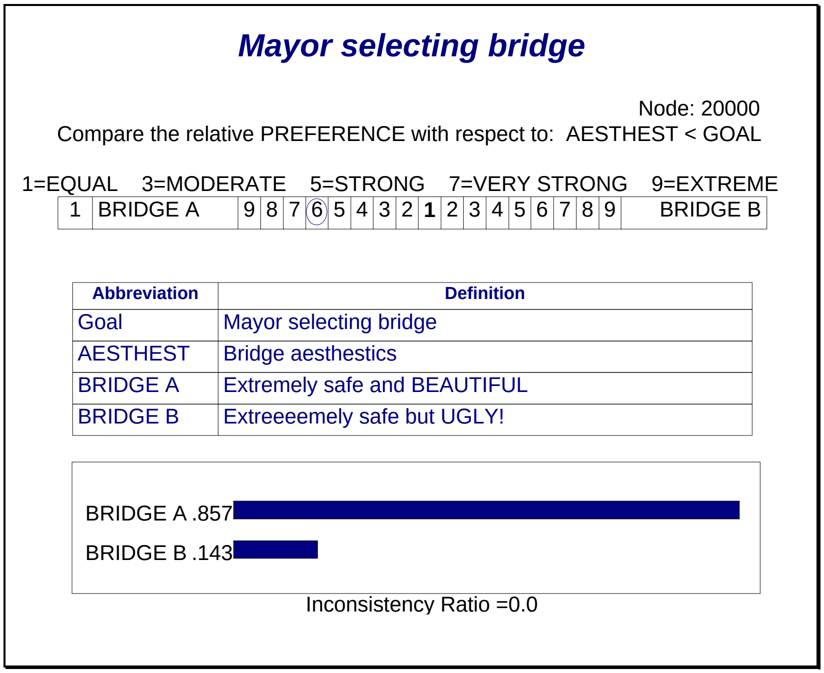

Figure 4 – Preference with respect to Aesthetics

Judgments about the preference for the bridges and resulting priorities are shown in Figure 3 and Figure 4. A synthesis using the top down approach is shown in Figure 5. This result, using a top down approach with no iteration, is counter-intuitive! Why should we choose an ugly bridge when we can choose one that is both safe and beautiful?

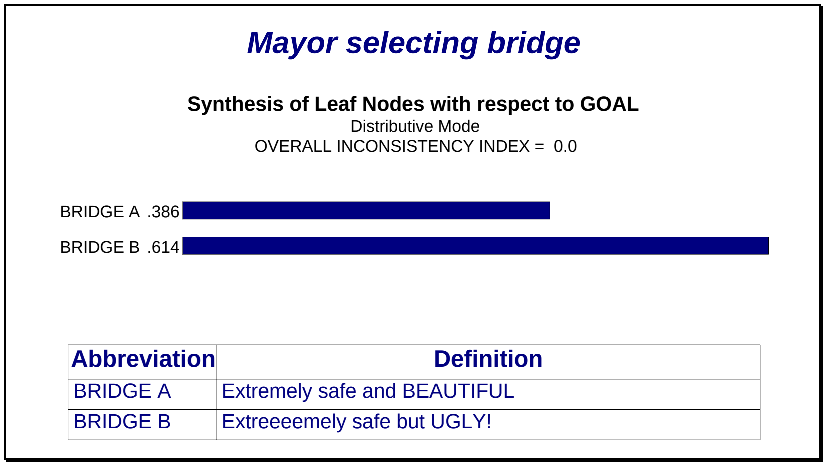

Figure 5 – Top Down Synthesis without Iteration

Top Down and Bottom Up

The 'top down' approach entails evaluating the importance of the objectives before evaluating the alternative preferences. A 'bottom up' approach, on the other hand, would consist of the evaluation of alternative preferences with respect to each objective before evaluating the relative importance of the objectives. If the decision-maker had used a bottom up approach instead, he/she would have learned that although design B is stronger than design A, both designs far exceed all safety standards. Furthermore, the decision-maker would have learned that design A is beautiful and while design B is ugly. Subsequently, while considering the relative importance of strength and aesthetics, the decision-maker might reasonably decide that aesthetics is more important than strength – see Figure 6. The resulting synthesis – see Figure 7, shows that Bridge A is now more preferable, a result that is also intuitively appealing.

This example illustrates that, if the decision-maker does not already know enough about the alternatives being evaluated, a bottom up approach will provide the necessary information so that reasonable judgments can be made about the relative importance of the objectives.

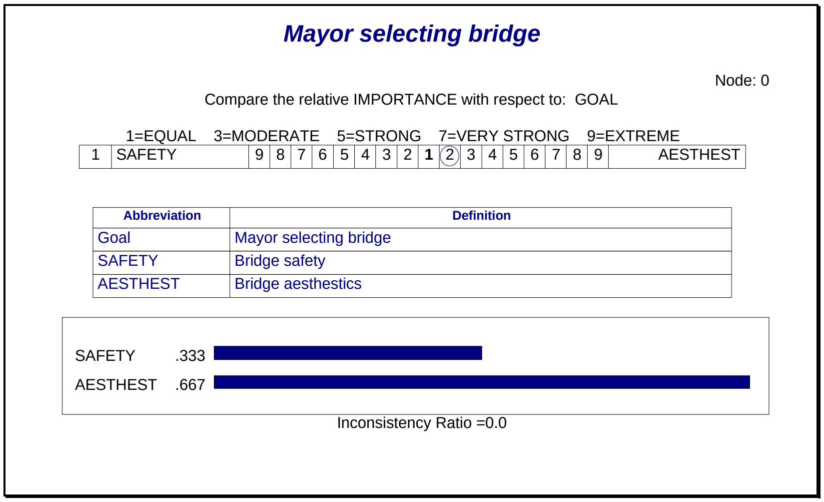

Figure 6 – Importance of Objectives with Bottom Up approach

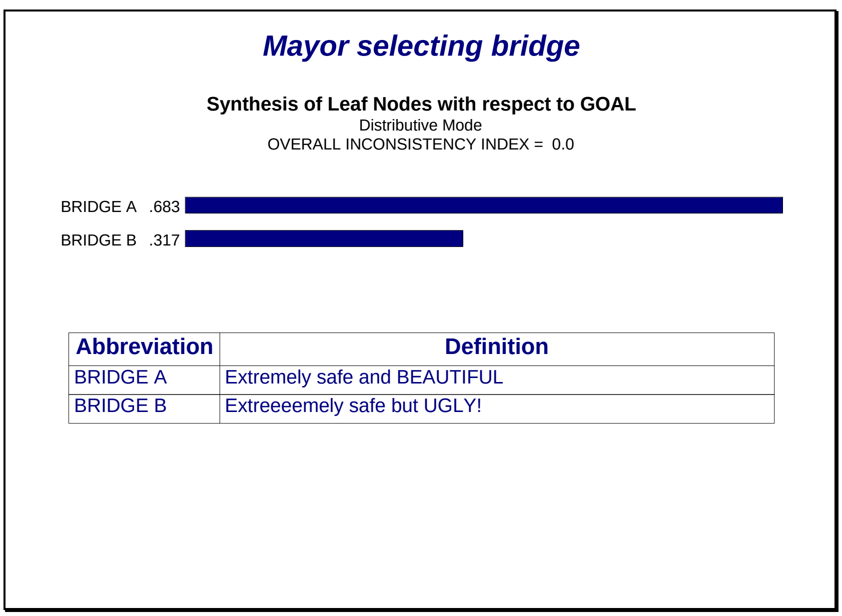

Figure 7 – Synthesis with Bottom Up approach

Even if a top down approach is used, no harm will result provided the decision-maker examines the tentative model results and questions its reasonableness.[2] In this example, the Mayor would, after synthesizing the first time, realize that the choice of the ugly bridge is counter-intuitive. Now knowing that both bridges are more than adequately safe he or she should re-evaluate his or her judgments. Doing so will result in the obviously correct choice of Bridge A.

AHP with Feedback –A more formal mechanism

A more formal approach is to use AHP with feedback[3]. An AHP model with feedback for this bridge selection example would, instead of asking the decision maker to compare the relative importance of safety and

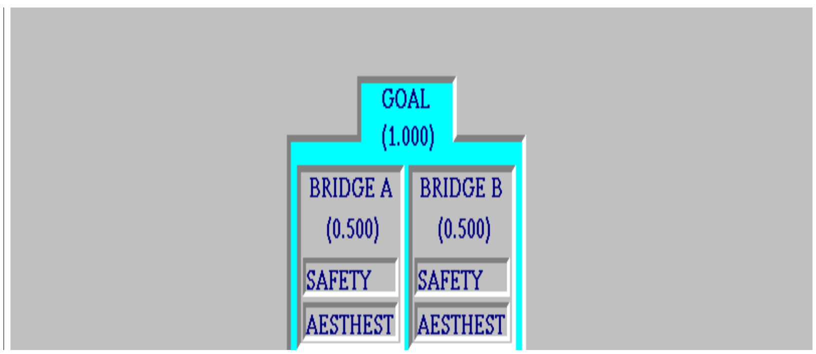

Figure 8 – Upside down Hierarchy

aesthetics with respect to the 'goal', instead ask for judgments about the relative importance of safety and aesthetics first with respect to Bridge A, and then with respect to Bridge B. We can think of this as turning the hierarchy in Figure 2 upside down, ignoring the priorities of the alternatives, and making judgments about the importance of the objectives with respect

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[2]As should always be done!

[3]Saaty, T.L., *Fundamentals of Decision Making and Priority Theory with The Analytic Hierarchy Process,* 1994, RWS Publications, Pittsburgh, PA., p38.

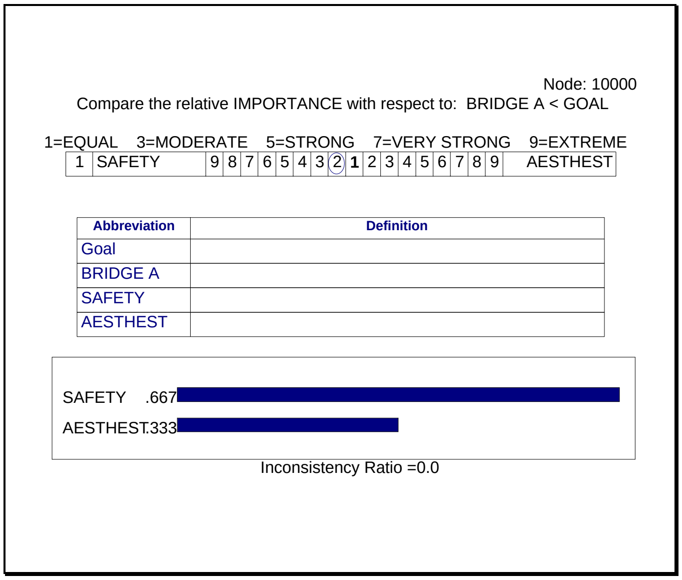

Figure 9 – Objective Priorities with respect to Bridge A

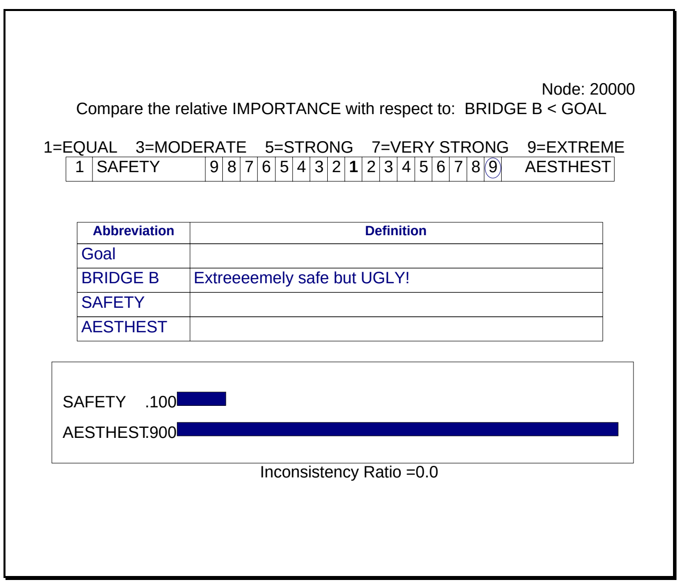

Figure 10 – Objective Priorities with respect to Bridge B

to each of the alternatives – See Figure 8. With respect to Bridge A, which is both safe and beautiful, a reasonable judgment is that safety is more important than aesthetics – see Figure 9.

However, with respect to Bridge B, which is safe but ugly, aesthetics is much more important than safety – see Figure 10.

Iterating for feedback

Figure 8 shows the model used to assess the relative importance of the objectives with respect to each alternative. Suppose we synthesized this model, with no judgments about the relative preference of the alternatives (or assuming that each alternative is equally preferable). The results are shown in Figure 11 – First Synthesis of Objectives with Respect to Alternatives.

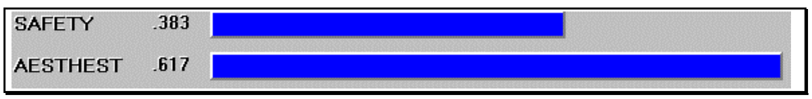

Figure 11 – First Synthesis of Objectives with Respect to Alternatives

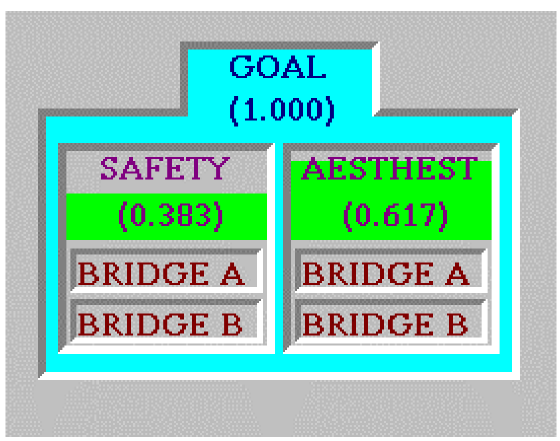

Figure 12 – Using Synthesized Objective Priorities from Dual Model

Figure 13 – First synthesis for Alternative priorities

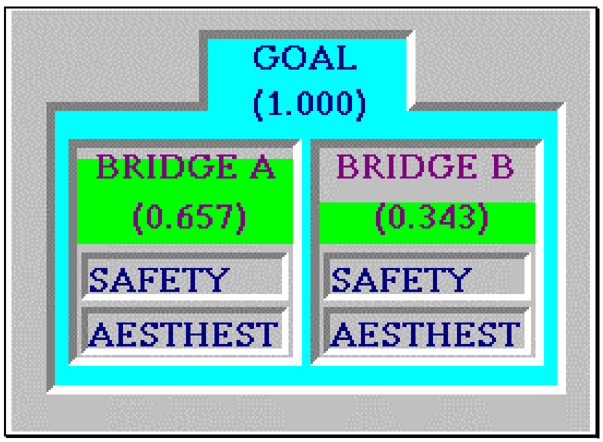

Figure 14 – Substituting first Synthesis Priorities

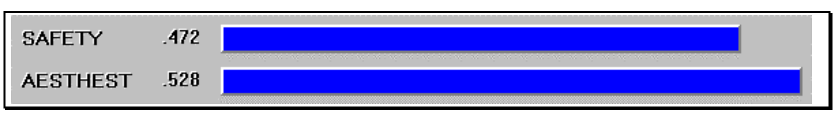

Figure 15 – Second Synthesis of Objective Priorities

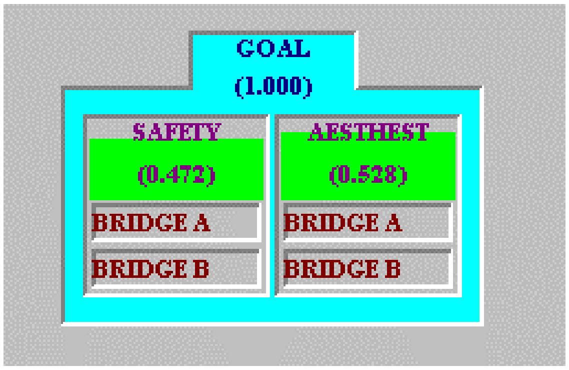

Figure 16 – Substituting 2nd Synthesis Priorities

Table 1 – Iterative Objective and Alternative Priorities

Iteration# / Alt or Obj	BridgeA or Safety	BridgeB or Aesthetics
2nd Alt:	.610	.390
3rd Obj:	.446	.554
3rd Alt:	.624	.376
4th Obj:	.454	.546
4th Alt:	.619	.381
5th Obj:	.451	.549
5th Alt:	.621	.379
6th Obj:	.452	.548
6th Alt:	.620	.380
7th Obj:	.451	.549
7th Alt:	.621	.379

By iteratively synthesizing the models in Figure 8 and Figure 12, each time replacing the priorities of the nodes below the goal with the priorities of the synthesized dual model, we converge on the priorities shown in Table 1. The alternative priorities derived above with a formal approach to feedback are similar to those derived using intuitive feedback (see Figure 7) where the judgments about the relative importance of the objectives were made with respect to the goal *after* examining the alternatives. The formal approach to feedback differs in that judgments about the relative importance of the objectives are made with respect to each alternative, rather than with respect to the goal. The iterations required for the formal feedback calculations can be carried out by a computer in the form of supermatrix calculations.

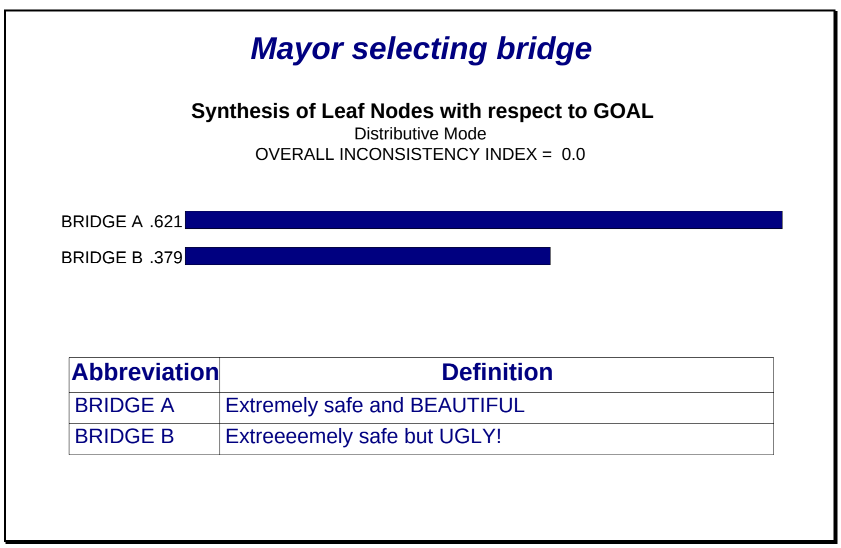

Figure 17 – Priorities after Iterating

Supermatrix for feedback

A supermatrix can be constructed and used to assess the results of feedback. Instead of iterating as we did above for illustrative purposes, the priority vectors for the alternatives with respect to each objective (from Figure 3and Figure 4) as well as the priority vectors of the importance of the objectives with respect to each alternative (from Figure 8 and Figure 9), are used to form a 'supermatrix' as follows:

0.	0.	0.667	0.1
0.	0.	0.333	0.9
0.333	0.857	0.	0.
0.667	0.143	0.	0.

The final priorities for both the objectives and alternatives are obtained by multiplying this matrix by itself numerous times[4] until the columns stabilize and become identical in each block[5]:

0.452	0.452	0.	0.
0.548	0.548	0.	0.
0.	0.	0.621	0.621
0.	0.	0.379	0.379

The objective priorities, represented in either of the first two columns, and the alternative priorities, represented in either of the last two columns, are the same as those achieved with iterative syntheses of the two models (Table 1). The supermatrix approach allows a great more deal of flexibility to incorporate feedback as we shall see shortly.

Intuitive versus formal feedback

It is possible to arrive at similar results using either a bottom up approach or top down followed by bottom up, where feedback occurs in the decision makers thought process, or a formal methodological approach

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[4] These calculations can be performed for feedback between adjacent levels using Team, or, more generally for any type of feedback using the ECNet software described later in this chapter.

[5] Saaty, T.L., *Fundamentals of Decision Making and Priority Theory with The Analytic Hierarchy Process,* 1994, RWS Publications, Pittsburgh, PA., p40.

where feedback is modeled in a supermatrix of priority vectors. Feedback is deeply ingrained in human functioning. Our ability to move from one part a room to another without falling over pieces of furniture (or even more remarkably to run, intercept and hit a moving tennis ball before it bounces twice within the confines of the court) rests in our brain's ability to continually process information based on cognition and our senses, and to give the appropriate commands to our muscles. Information is continually fed back so that, for example, adjustments to the current path are made based on our desires about our destination, present position, obstacles in our path, forecast of what will happen, and so on. Halfway across the room we might decide to change our destination realizing that the chair we had started out for will likely be taken by another person by the time we arrive. There is no question that humans can mentally process information incorporating feedback. Our ability to make judgments about the importance of objectives based on our knowledge of alternatives is an example of such feedback. However, there are also situations where we can benefit with a decision aid that formally incorporates feedback. An increased understanding of what our minds can do easily and what we find difficult will be important so that we can employ the proper balance of cognition and decision aids. Our ability to catch a ball, or (for some humans) to think several moves ahead in a game of chess is truly remarkable. On the other hand, psychologists have shown that the human mind has very limited abilities. We function very well without decision models for the vast majority of our decisions. Yet our everyday decision rules or common simplistic strategies are often not adequate for making crucial decisions. In the example presented above, our intuition is more than adequate in selecting the best of two bridges given the two alternatives and two objectives of safety and aesthetics. However intuition alone would not be adequate if there were several alternatives and many tradeoffs to consider involving perhaps ten, twenty, or fifty objectives. We need to continue to investigate and learn more about human abilities and limitations in making complex decisions so that we can provide decision support where it is needed and in ways that best augment, rather than replace human thinking. The Analytic Network Process (ANP) is a step in that direction.

The Analytic Network Process

Saaty has extended the Analytic Hierarchy Process to incorporate various types and degrees of feedback – a process referred to as the Analytic Network Process or ANP[6]. The ANP is the first mathematical theory that makes it possible for us to deal systematically with all kinds of dependence and feedback. The reason for its uniqueness is the way it elicits judgments and uses measurement to derive ratio scales. Priorities measured on ratio scales are necessary for performing the basic arithmetic operations of adding within the same scale and multiplying different scales meaningfully as required by the ANP. The ANP provides a framework of clusters of elements connected in any desired way to investigate the process of deriving ratio scales priorities from the distribution of influence among elements and among clusters. The distribution of influence is represented by interactions and feedback within clusters (inner dependence) and between clusters (outer dependence). The AHP is a special case of the ANP. Although some decision problems are best studied through the ANP, it is not true that forcing an ANP model always yields better results than using the hierarchies of the AHP. There are examples to justify the use of both. We have yet to learn when the shortcut of the hierarchy is justified, not simply on grounds of expediency and efficiency, but also for reasons of validity of the outcome.

The ANP is implemented in the software ECNET[7] and is a coupling of two parts. The first consists of a control hierarchy or network of criteria and subcriteria that control the interactions in the system under study. The second is a network of influences among the elements and clusters. The network varies from criterion to criterion and a supermatrix of limiting influence is computed for each control criterion. Finally, each of these supermatrices is weighted by the priority of its control criterion and the results are synthesized through addition for all the control criteria.[8] We will

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[6] Saaty, T.. L. *Decision Making with Dependence and Feedback, The Analytic Network Process*, 1996, RWS Publications, Pittsburgh, PA.

[7] ECNET was developed jointly by Ron Chan and Thomas Saaty.

[8] In addition, a problem is often studied through a control hierarchy or system of benefits, a second for costs, a third for opportunities, and a fourth for risks. The synthesized results of the four control systems are combined by taking the quotient of the benefits times the opportunities to the costs times the risks to determine the best outcome.

deal only with the second part right now and take as our only control criterion the benefits to be derived from the alternatives under consideration.

A Car buying example with Feedback

Consider a car buying example where the objectives are Initial Cost, Repair Costs, and Durability and the alternatives are American cars, Japanese cars and European Cars. A traditional hierarchy for this problem is shown in Figure 18.

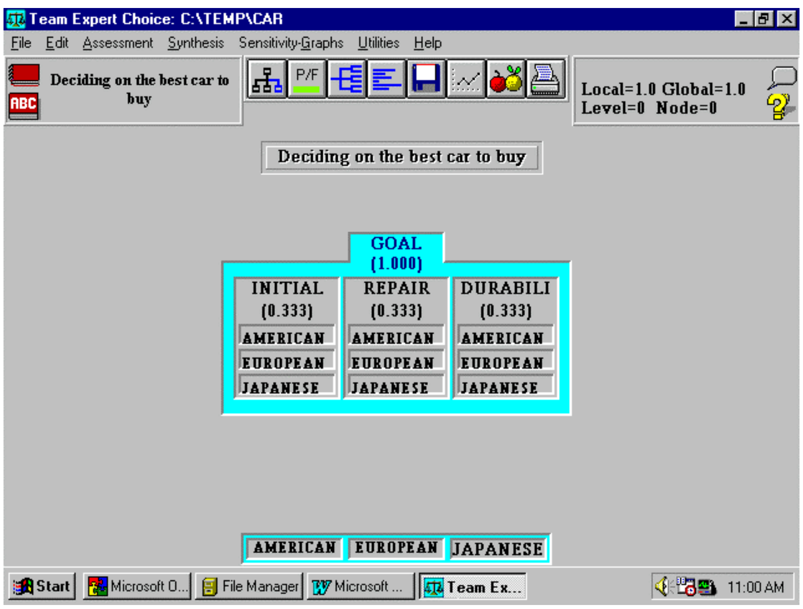

Figure 18 – The First Car Model Hierarchy

An upside down hierarchy

Instead of making judgments about the relative importance of the objectives with respect to an ‘overall goal’, it might be more meaningful (although more time consuming) to make such judgments with respect to each of the alternatives. In essence, you have turned the first hierarchy upside down. The model is shown in Figure 19. For example, as of the early 1990’s, when you thought of American cars, the low initial cost would have been more important than durability. On the other hand, when comparing the objectives with respect to European cars, durability would have been more important than initial cost. And when you thought of Japanese cars, requiring fewer repairs would have been more important than initial cost or durability. The result of this model is the prioritization of the objectives.

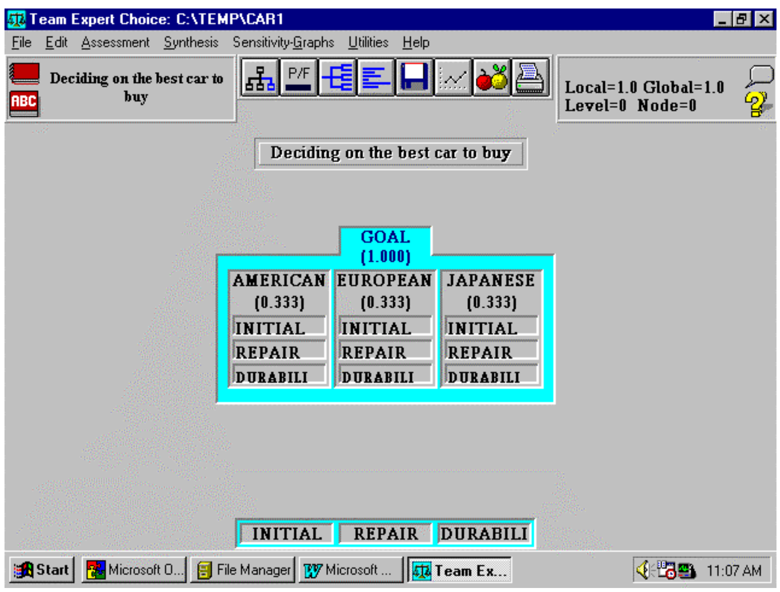

Figure 19 – The Second Car Model Hierarchy (Upside-down)

Combining the Two Hierarchies in a Feedback Network Model

The implicit interactions among factors in the two hierarchies above is what we mean by feedback and dependence. Just as we iterated between two related models in the bridge example earlier, we can do the same for this car buying example. In general, however, we can best handle these considerations by modeling the problem as a network. What if one currently owns a particular kind of car (American, European, or Japanese), would that not somehow influence our perceptions, and hence judgments about, repair costs and durability? Those judgments may in turn may cloud our judgments about the various manufacturers.

Influences in hierarchies only flow in one direction--downward. The 'Goal' of a traditional hierarchy can be thought of as a 'source' of water. The water is distributed to the objectives, then to the sub-objectives, and so on, down to the alternatives. The relative amount of water collected by each alternative determines the alternative's priority. Water never flows 'out' of the alternatives in the traditional hierarchical model.9 The feedback network model, however, differs in that water flowing into the alternatives also flows out. The water flowing out can be 'fed back' to the objectives (as well as other elements in a more elaborate network). The 'steady state' water flow in a network is continuous – water flows out of the objectives, into the alternatives, then flows out of the alternatives into the objectives. There is no need for a 'goal' node as a source of water because the alternatives no longer act as absorbing 'sinks'.

Network models do not have levels such as goal, objectives, and alternatives. Instead, the elements (or nodes) in a network model are grouped into clusters, such as an objectives cluster and an alternatives cluster. The feedback model for this car example has two clusters:

OBJECTIVES containing the objectives Initial Cost, Repair Cost and Durability.

ALTERNATIVES containing the alternatives American, Japanese and European.

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[9] The 'supermatrix' representation of a traditional hierarchy must have an identity matrix in that portion of the matrix representing the alternatives to indicate that the alternatives are 'sinks' for what flows in.

Links in a Feedback Network Model

The flow of influence in a feedback network model is specified by links. A link from one element, such as an objective, to other elements, such as alternatives, specifies that influence can flow from the former to the latter, and that pairwise comparisons will be made to indicate the relative amount of influence that flows from the former to each of the latter. Conversely, when pairs of elements can be meaningfully compared with respect to another element, then a link from the latter to the former is appropriate.

One way to identify ALL possible links is to consider each element and identify all other pairs of elements that can logically be compared with respect to the element being considered. This approach can lead to a very complex structure that might take an inordinate amount of time to evaluate. Another approach is to add links only for those situations where influence is apparent.

Each objective in the car network example has a link to the three alternatives to indicate the flow of influence from the objective to the alternatives. Pairwise comparisons will be made to determine the relative influence that the objective has on the relative preferences of the alternatives.

Similarly, each alternative in the car network example has a link to the three objectives to indicate the flow of influence from the alternative to the objectives. Pairwise comparisons will be made to determine the relative influence that the alternative has on the relative importance of the objectives.

Clusters themselves become linked when elements within them are linked. When a cluster is linked to more than one other cluster, comparisons will also have to be made for the relative influence of the latter clusters on the former.

Two Kinds of Questions to Answer in Making ANP Comparisons

When making pairwise comparisons in an ANP model the questions are formulated in terms of dominance or influence. When comparing a pair of elements in one cluster with respect to an element in another (or the same) cluster, we ask either:
Which element of the pair has greater influence?, or
Which element or the pair is influenced more?
The same type of question should be used throughout the evaluation.

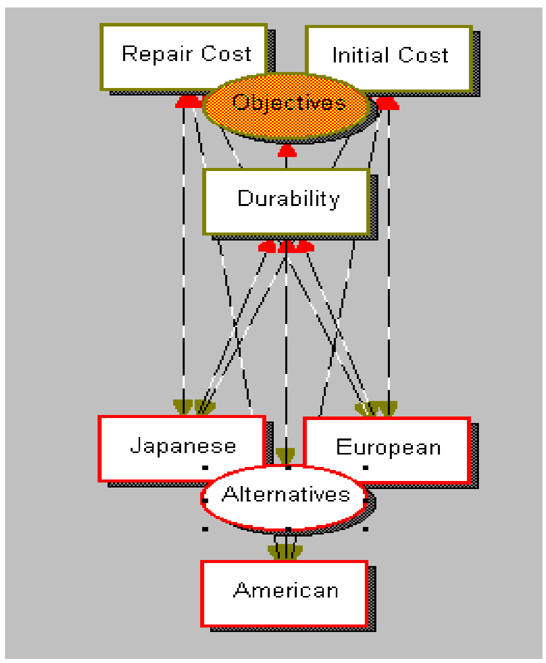

Figure 20 – Cluster Centric View of Car Network

The clusters, elements and links for the car example is shown in Figure 20 and Figure 21, the former a cluster centric view and the latter an orbital

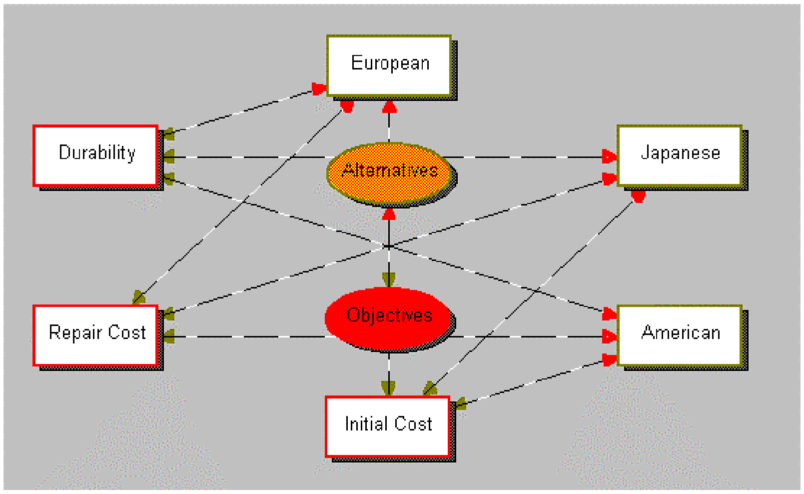

Figure 21 – Orbital View of Car Network

view. A model is said to have outer dependence when elements are linked to elements in another cluster. A model also has inner dependence when elements are linked to other elements in their own cluster. There is no inner dependence in this example.

In the ANP, just as in the AHP, we are usually looking for a prioritization of alternatives as a result. Thus, in general, every feedback model should include a cluster of elements that will be our alternatives.

Summary of Steps in Building a Feedback Network

We have to:

identify the clusters as they relate to the problem
identify the elements within each cluster
identify dependencies among the elements and link them
elicit judgments on the elements
elicit judgments on the clusters (if necessary) synthesize the result